On Prepayment and Rollover Risk in the U.S. Credit Card Market

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ABSTRACT

Prepayment risk is a major consideration in the US credit card market. According to industry studies, about 17% of balances are transferred annually. Using a dynamic model of repricing, this paper analyzes the impact of prepayment risk on the functioning of the credit card market. We show that the model can account for the level of balance transfers in the data and is also consistent with the observed change in the cross-sectional dispersion of credit card interest rates, which thus far has been attributed to a more precise pricing of default risk. Importantly, we show that the very features that allow our model to account for the cross-sectional features of the market unveil a novel source of macroeconomic fragility. We quantify the potential of this new channel to account for the observed deleveraging in the credit card market during the 2007-09 financial crisis.

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1 Introduction

One of the key features of the US credit card market is that contracts are subject to a large risk of debt prepayment. Over the 90s the volume of repriced debt rose rapidly and, according to industry studies, in the early 00s as much as 17% of credit card balances were transferred annually by consumers seeking better terms (Evans and Schmalensee, 2005). Today almost every credit card company tries to poach the customers of other companies by offering them a balance transfer option at low interest rates. Yet, very little is known about the impact of this peculiar feature of the data given that existing models of unsecured consumer credit counterfactually assume that credit contracts are exclusive and only subject to default risk.[1]

To fill this gap, our paper develops a dynamic model of unsecured credit featuring non-exclusive credit lines and dynamic repricing. Our key point of departure from existing theories of unsecured credit is that the relationship between borrowers and incumbent lenders endogenously evolves over time and, due to the possibility of prepayment, is critically shaped by changes in lenders' exposure to borrower default. This is because entry of future lenders and thus potential balance transfer offers are conditional on information updates about borrower time-varying credit worthiness.

In our model, prepayment risk affects credit contracts exposed to default in two important ways. First, incumbent lenders may choose to increase interest rates to secure enough revenue to cover both prepayment and default risk. We refer to this type of credit cards as 'rollover contracts', as they actually allow for full prepayment in the form of a balance transfer to a new credit card. The interesting feature of these particular contracts is that they introduce rollover risk from borrowers' perspective, who find their rates too onerous for long term borrowing and thus only accept them in expectation of a future low interest rate balance transfer offer. The presence of rollover risk can render the market vulnerable to short-lived credit supply shocks, despite the long maturity that characterizes credit card contracts. Alternatively, incumbent lenders may choose to substantially expand their offered credit limits. Such strategy may reduce the size of future balance transfers by pushing borrowers closer to *strategic default*,

[1] To account for default related statistics, existing models assume a relatively long baseline period length, which is 1-5 years long. Contracts deterministically expire after this time period and need to be repriced. In contrast, in our model entry is optional and incumbent contract duration is endogenous. See Livshits, McGee and Tertilt (2007), for example.

i.e., the possibility that consumers have so much access to credit combining all their credit cards that they over-borrow and choose to default also in 'good times'. These 'crowding out contracts' do not create rollover risk, but instead lead to high default losses in equilibrium, which lenders trade-off against the benefit of locking in borrowers.

We use our model to quantitatively account for the following three key features of the US credit card market: 1) a relatively high gross level of unsecured debt (9% relative to average disposable income), 2) a significant fraction of debt being lost due to consumer defaults (5.5% of outstanding balances), and, finally, 3) a high volume of balance transfers (20% per annum). In order to do so we propose a life-cycle model of consumer bankruptcy, along the lines of the framework developed by Livshits, McGee and Tertilt (2007). Our model perfectly matches all three facts and, most importantly, it suggests that 3) is only consistent with a substantial presence of rollover contracts. Their prevalence is also consistent with the fact that the dispersion of interest rates in the credit card market has been rising in the data. Compared to the counterfactual exclusivity regime featuring no prepayment risk, which our model nests, the interest rate distribution is more spread out in the presence of prepayment risk under non-exclusivity. In particular, it exhibits a fatter right tail due to the high rates in rollover and crowding out contracts and a fatter left tail brought by the low introductory rates associated to balance transfer offers. Figure 1 shows that, in parallel to the exponential rise of balance transfers, this is precisely what happened to the distribution of credit card rates on outstanding balances in the 90s and over the 00s.

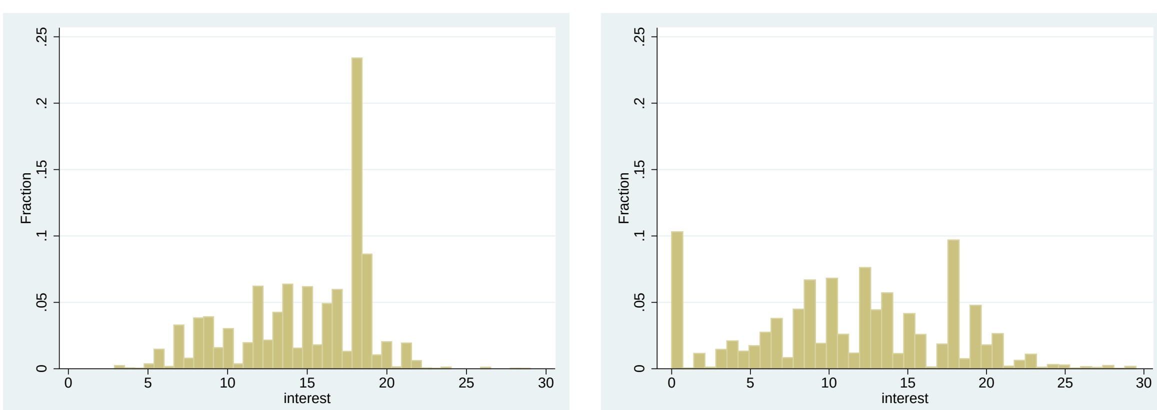

Figure 1: Rate Distribution: 1995 (left) and 2004 (right) [Survey of Consumer Finances].

The equilibrium contracts giving rise to the observed volume of balance transfers can have important implications beyond explaining the cross-sectional features of the market. In particular, they can introduce fragility to aggregate credit supply disruptions, which we explore in the context of the substantial deleveraging experienced by the US credit card market during the most recent financial crisis. As is well known, the credit card market has been significantly affected by the crisis: the default rate increased by a factor of two between 2008 and 2010, and there has been a significant decline in credit card debt outstanding (credit card debt to income fell by 10%), both due to elevated default rates as well as consumers actually reducing their balances. While it is not entirely clear to what extent the latter development was supply rather than demand driven, we consider credit supply shocks to illustrate how our model can explain the observed deleveraging. Nevertheless, two facts suggest that supply shocks likely played a significant role: the steep decline in new credit card accounts offering promotional balance transfers (a 70% drop) and, tellingly, the sudden drop in overall credit card mail solicitations (58%), despite a simultaneous increase in response rates between 2007 and 2010. If anything, one should expect a fall in response rates were the decline in solicitation demand driven.[2]

Motivated by the above evidence, we use the crisis episode to show that, by introducing rollover risk, equilibrium contracts can significantly contribute to debt deleveraging. The rationale goes back to the very nature of rollover contracts. Recall that they arise because consumers expect to refinance their debt in the short run. If such refinancing opportunities suddenly dry up, as could have been the case during the notorious credit crunch, consumers who are stuck with those contracts will try to quickly reduce their balances. Our model fully accounts for both the substantial reduction in credit card debt to income and the increase in interest rates on outstanding credit card debt.

More precisely, we model the impact of the crisis on the credit card market by (unexpectedly) increasing the probability that consumers fall into financial distress (*macroeconomic shock*) and by partially disrupting dynamic entry (*credit crunch*). The former is calibrated

[2]Response rates to direct mail solicitations went from 0.467% in 2006 to 0.575% in 2009 (data from Synovate Mail Monitor). The data in new accounts with a balance transfer promotion is from Argus, Information & Advisory Services, LLC (`files.consumerfinance.gov/f/2011/03/Argus-Presentation.pdf`) Data on direct mail solicitations is from Mintel Media (`www.comperemedia.com`).

to match the spike in the charge-off rate, i.e., the ratio of defaulted to outstanding credit card debt. The credit crunch, on the other hand, which in our model is very short-lived and only disrupts repricing opportunities, explains the change in outstanding interest rates and causes a further drop in debt-to-income of about 16% of the total deleveraging in the model. Incidentally, while our model implies an increase in the average interest rate, the effect is asymmetric across agents. Only about 11% of revolvers are affected by interest rate hikes, but in their case the increase in interest rates typically varies between 10-20 percentage points. Such asymmetric effect amplifies the impact of the credit crunch on consumer welfare.

A remarkable aspect of our quantitative exercise is that deleveraging in the data is matched by only disrupting repricing opportunities, without affecting consumers' overall access to credit. In contrast, a model with exclusive contracts and no prepayment risk, which we study for comparison, falls short of matching the deleveraging and misses the increase in interest rates since the above disruption of credit has no effect in the selection of contracts under exclusivity. We believe this to be one of the main appeals of our approach given the relatively long maturity of credit card contracts in the data, and the fact that banks after 2009 were no longer allowed to force repayment nor increase interest rates on pre-existing debt. In this context, a short-lived credit shock would not severely affect access to pre-existing credit cards, rendering a change in households' time preferences or in their long term economic outlook as the only plausible alternatives to explain the deleveraging experienced during the crisis. Accordingly, by showing how the nature of credit card competition leaves the unsecured credit market prone to deleveraging in response to credit shocks, our paper studies a potentially important transmission channel that might have exacerbated the crisis.

In contrast to our paper, existing theories on non-exclusive lending only consider static entry — see DeMarzo and Bizer (1992), Petersen and Rajan (1995), Kahn and Mookherjee (1998), Parlour and Rajan (2001), Bisin and Guaitoli (2004), Hatchondo and Martinez (2007), and also the information sharing model of Bennardo, Pagano and Piccolo (Forthcoming). In addition, existing quantitative models of unsecured borrowing such as Livshits, McGee and Tertilt (2007) and Chatterjee et al. (2007) assume an exclusivity setup in which consumers issue a state-non contingent bond with one-period maturities.[3] An exception is the model

[3]See also the relevant work of Narajabad (2012), Athreya, Tam and Young (2012), and Sanchez (2012).

of Rios-Rull and Mateos-Planas (2007), which exhibits long-term credit lines and consumers can switch to a new credit line although they are not allowed to hold multiple contracts (exclusivity). In this regard, we offer a quantitative model that analyzes the impact of a non-exclusive contractual environment in which debt maturity is (partially) endogenous and state dependent.

2 The Model

We develop in this section an analytic model of the credit card market. We later extend this model to a multi-period life-cycle environment that we calibrate to the US data. While several aspects of the setup laid out here are intentionally simplified, our life-cycle model is more general and fully dynamic.

2.1 Environment

The economy is comprised of a large number of risk neutral lenders and risk averse consumers and there are two periods. Lenders extend unsecured credit lines to consumers, which they use to smooth consumption intertemporally and/or absorb financial distress shocks. Lenders compete in two rounds of Bertrand competition, where the first round takes place before consumers know whether they are under financial distress, while the second round takes place after both consumers and lenders learn the future realization of the distress shock.[4] Contracts are pre-committed to consumers but non-exclusive. That is, during the second round consumers do not have to cancel their initial credit lines, which allows them to smoothly roll over part of their debt onto a possibly cheaper credit line through a balance transfer. This key aspect of our model is intended to capture the effect of a highly developed credit reporting system in the US, which on one hand facilitates competition by stripping incumbent lenders from informational advantages (Bertrand competition in each round), but on the other hand gives rise to repricing of existing debt. We also assume that the second round is subject to

[4]Our specification assumes that consumers accept one contract per round. The following simple game can rationalize this assumption as an equilibrium outcome: within each round of competition, borrowers shop sequentially for lines while lenders observe all contracts being accepted in the process and can change the terms of unaccepted offers. Lenders can commit to not change terms, unless the borrower applies for more credit within the same round. Under reasonable assumptions, merging two same-round lines into one is always better, as it reduces the marginal interest rate while providing the same aggregate limit.

an exogenous delay, and thus our model nests the standard exclusivity regime when delay is equal to one period.

2.2 Consumers

Consumer preferences are given by $u(G(c_1, c_2))$, where c_i denotes consumption in period i, u is a concave utility function and G is a quadratic intertemporal aggregator.

The timing of events from the consumer's point of view is as follows. After accepting the first credit card, denoted by $\mathcal{C}$, and characterized by interest rate R and credit limit L, consumers learn whether they will be exposed in the second period to an idiosyncratic financial distress shock (e.g. medical bills, divorce, unwanted pregnancy).[5] This information is relayed, possibly with some delay, to lenders and the second round of competition opens up. In this new round borrowers can accept a balance transfer offer $\mathcal{C}'$, given by a rate R' and limit L'. With both contracts on hand, consumers choose how much to borrow b and how much to consume in the first period, and any potential balance transfers are executed. At the end of the period consumers default or pay back their debts, which determines their second period consumption. This timing of events makes the model more tractable by avoiding taking expectations to pin down the optimal borrowing policy. Since the probability of distress is low, this does not make a big difference qualitatively. In what follows, we consider interest rates satisfying $R > R'$ (given our assumptions below and lender behavior in the second round this is without loss of generality).

Interest rate on borrowing is distributed among lenders proportionally to the delay associated to the second round, which we parameterize by $\zeta \in [0, 1]$. Specifically, the accrued interest on transferred debt that corresponds to the first round line is proportional to $(1 - \zeta)$. In the absence of delay ($\zeta = 1$), all revenue on transferred debt is undercut while contracts are fully exclusive and thus not subject to repricing when $\zeta = 0$. In the latter case, our model features no prepayment risk, and is thus similar to existing theories of unsecured credit.

In each period, consumers receive deterministic income $y > 0$ and, as already mentioned, face an i.i.d. binary expense shock $\kappa \in \{0, x\}$, $x > 0$, which hits consumers in period two with

[5] We assume that agents perfectly anticipate the shock realization to gain analytical tractability but it plays no substantive role otherwise.

probability p. They start with some pre-existing debt $B > 0$ (endogenous in the life-cycle model), and smooth consumption across periods by resorting to credit card borrowing and an option to default. Default involves an exogenous pecuniary penalty.

Intertemporal consumption preferences are represented by a quadratic aggregator function $G(c_{1,\kappa}, c_{2,\kappa}) = c_{1,\kappa} + c_{2,\kappa} - \mu(c_{2,\kappa} - c_{1,\kappa})^2$, where $c_{i,\kappa}$ represents consumption in period i given shock realization κ and $\mu > 0$ is a curvature parameter. The quadratic function is used here merely for convenience, as it greatly facilitates analysis. In particular, it leads to optimal borrowing being linear in the marginal interest rate.[6]

The borrowing constraint that consumers face in our model captures the predatory nature of balance transfers. Specifically, we assume that consumers cannot borrow more than L in the first period, and so the initial line $\mathcal{C}$ is essential to allow them to rollover their pre-existing debt B into the period. Hence, consumers can only use the second round line to reduce interest payments due on the initial line by transferring debt to the new line. In addition, they can also max out and default on both cards in the second period, thereby exposing both lenders to default risk.

Formally, we can think of consumers as planning their future default decision $\delta(\kappa, \mathcal{C}, \mathcal{C}') \in \{0, 1\}$, with $\delta = 1$ implying default, and choosing borrowing level $b(\kappa, \mathcal{C}, \mathcal{C}')$. This choice must maximize their indirect utility function given by

$$V(\kappa, \mathcal{C}, \mathcal{C}') \equiv \max_{\delta \in \{0,1\}} V^{\delta}(\kappa, \mathcal{C}, \mathcal{C}'). \tag{1}$$

where $V^{\delta}(\kappa, \mathcal{C}, \mathcal{C}') = u(G(c_1, c_2))$ stands for the indirect utility function *conditional* on default decision δ, which we characterize next.

Repayment. In case of no default ($\delta = 0$), the budget constraints are, respectively,

$$c_{1,\kappa} + B + \rho(b, \mathcal{C}, \mathcal{C}')/2 = y + b, \text{ with } b \leq L, \tag{2}$$

[6]Qualitatively similar results can be derived for G is a CES aggregator and are available from the authors upon request.

and

$$c_{2,\kappa} + \rho(b, \mathcal{C}, \mathcal{C}')/2 + \kappa + b = y,$$

where $\rho(b, \mathcal{C}, \mathcal{C}')$ denotes interest payments, which are evenly spread across the two periods.[7] Such payments take into account balance transfers that are aimed to reduce the overall interest burden, subject to delay $1 - \zeta$. That is, as already mentioned, the budget constraint assumes that the first round lender receives a fraction $(1 - \zeta)$ of the interest due on the full balance (b) plus a fraction ζ of the interest due on the *residual* balance $\max\{0, b - L'\}$. Accordingly, interest payments, given borrower beliefs about $\mathcal{C}'$, are given by

$$\rho(b, \mathcal{C}, \mathcal{C}') = \begin{cases} R[\zeta(\max(b - L', 0) + (1 - \zeta)b] + \zeta R' \min(L', b) & b > 0 \\ 0 & b \leq 0, \end{cases} \tag{3}$$

where $b < 0$ implies saving. Note here that, given that wlog $R > R'$, the decision to transfer balances is mechanical: the consumer will transfer as much debt from $\mathcal{C}$ to $\mathcal{C}'$ as possible, i.e., up to L'.

To simplify notation, let b_k denote $b(\kappa, \mathcal{C}, \mathcal{C}')$. By the first order condition, applicable whenever L is non-binding, borrowing is given by

$$b_\kappa = \frac{B - \kappa - \rho_b(\mathcal{C}, \mathcal{C}')/4\mu}{2} \tag{4}$$

where ρ_b stands for the partial derivative of ρ with respect to b, i.e., the marginal interest rate. In this case, the marginal interest rate is equal to R whenever the balance transfer is *incomplete*, and $(1 - \zeta)R + \zeta R'$ when the initial line is subject to a *full* balance transfer. The borrowing policy implies an interest rate distortion of intertemporal consumption smoothing given by the quadratic term $\mu(c_{2,\kappa} - c_{1,\kappa})^2 = \mu\left(\frac{\rho_b}{4\mu} - \kappa\right)^2$.

Default. Under default, the consumer can discharge both the principal and the interest due. Thus, the first period budget constraint takes the form

$$c_{1,\kappa} + B = y + b;$$

[7]This is done for tractability and has no substantive bearing on our results.

while the second period budget constraint reflects the fact that, subject to a default penalty proportional to income given by $(1-\theta_\kappa)y$, the consumer can max out on both credit lines prior to discharging all credit card debt as well as a fraction $1-\phi$ of the distress shock:

$$c_{2,\kappa} \; + b = \theta_\kappa y - \phi\kappa + L + L',$$

Here we make the natural assumption that income after defaulting, net of shocks, is higher in normal times than in distressed times: $\theta_0 y \geq \theta_x y - \phi x$. As we comment below, such restriction does not affect our qualitatively results but simplifies the exposition.

The above setup implies that the default decision is effectively governed by a comparison of intertemporally aggregated consumption along each path (default/repayment), both endogenously determined in equilibrium. For this reason, it is relatively difficult to characterize. Nevertheless, we can show that, given any fixed set of contracts, punishment for defaulting induces a well-defined bound on consumers' aggregate access to credit $L + L'$, which we call the borrower's *credit capacity*. Above this bound, the consumer decides to default even if she is non-distressed, and below this level she repays when she is not hit by the shock. Notice that a low capacity, e.g., due to small default penalties, can severely constrain credit. On the other hand, it can also serve as a protection against balance transfers from the perspective of first round lenders. This is because it mitigates the exposure of initial contracts to prepayment risk by limiting the residual capacity that second round lenders can utilize. The existence of a finite credit capacity is irrespective of whether default penalties are pecuniary (our focus on pecuniary penalties is for analytical convenience). Furthermore, as the next lemma shows, the capacity under $\kappa = 0$ decreases w.r.t. L at least one-to-one, implying that higher L 'crowds out' future balance transfers (L') by more than one-to-one.

Definition 1. *(credit capacity) Given* $(L, R, R') \geq 0$*,* $\mathcal{L}_{max}(\kappa; L, R, R')$ *represents the total credit limit such that* $V^0(\kappa, \mathcal{C}, \mathcal{C}') < V^1(\kappa, \mathcal{C}, \mathcal{C}')$ *for all* $L' + L > \mathcal{L}_{max}(\kappa; L, R, R')$ *and* $V^0(\kappa, \mathcal{C}, \mathcal{C}') \geq V^1(\kappa, \mathcal{C}, \mathcal{C}')$ *otherwise.*

Lemma 1. $\mathcal{L}_{max}(\kappa; L, R, R')$ *is bounded for all* L*, and decreasing in* L *for all non-binding* L*.*

Unless otherwise noted, all proofs are in the Appendix. To ease notation, we will write

$\mathcal{L}_{max}(L, R)$ to denote the capacity associated to $\kappa = 0$ and $R' = 0$.

2.3 Lenders

Lenders maximize expected profits, and compete in a Bertrand fashion. They have deep pockets and their cost of funds is normalized to zero. When a consumer defaults, because she maxes out on available credit limits, lenders who extended credit to this consumer incur a loss equal to the size of their credit line. Specifically, for any set of contracts $\mathcal{C}$ and $\mathcal{C}'$ obeying $R' < R$, the profit function of initial lenders is given by

$$\pi(\kappa, \mathcal{C}, \mathcal{C}') = (1 - \delta(\kappa, \mathcal{C}, \mathcal{C}'))R[\zeta \max\{b_\kappa - L', 0\} + (1 - \zeta) \max\{b_\kappa, 0\}] - \delta(\kappa, \mathcal{C}, \mathcal{C}')L, \quad (5)$$

while the profit function of second round lenders is given

$$\pi'(\kappa, \mathcal{C}, \mathcal{C}') = (1 - \delta(\kappa, \mathcal{C}, \mathcal{C}'))\zeta R' \min\{L', b_\kappa\} - \delta(\kappa, \mathcal{C}, \mathcal{C}')L'. \quad (6)$$

The above profit functions highlight an important implication of non-exclusivity of contracts from initial lenders' point of view. Namely, under non-exclusivity a higher credit limit L can reduce the size of future balance transfers given by $\mathcal{L}_{max}(.) - L$, thereby partially protecting interest revenue.[8] Importantly, this crowding out is only present under *incomplete* balance transfers, i.e., for contracts satisfying $b_0 > L_{max} - L$. Otherwise, a small increase in L only increases default exposure without affecting interest revenue. This distinction will be important for our results. It will lead us to conclude that there are two possible types of credit lines exposed to default in equilibrium. Since competition is dynamic and takes place in two rounds, before and after the resolution of uncertainty on the consumer side, the equilibrium in the credit market is required to be subgame perfect. Applying backward induction, we solve first the problem of second round lenders.

Ex-post lending market (second round). Bertrand competition implies that equilibrium contracts must maximize consumer's indirect utility subject to zero (expected) profits.

[8]Recall that, by Lemma 1, $\mathcal{L}_{max}$ is decreasing in L.

Recall that second round lenders learn the future shock realization before offering their contract. Accordingly, initial lender's choice is contingent both on κ and on $\mathcal{C}$:

$$\mathcal{C}'(\mathcal{C},\kappa) = \arg\max_{\mathcal{C}'} V(\kappa,\mathcal{C},\mathcal{C}'), \text{ subject to } \pi'(\kappa,\cdot) \geq 0.$$

Given this, it is clear that the second round lenders best respond to $\mathcal{C}$ under Bertrand competition by charging the cost of funds interest rate and extending credit up to credit capacity for any given borrowing level. This result is summarized in the lemma below. [9]

Lemma 2. $\mathcal{C}'(\mathcal{C},\kappa) = (0, \max\{0, \mathcal{L}_{max}(\kappa; L, R, 0) - L\})$ *for all* $\mathcal{C} = (L, R)$.

Proof. Omitted. □

Initial lending market (first round). Anticipating $\mathcal{C}'(\cdot, s)$, lenders choose first round contract $\mathcal{C}^* = (R^*, L^*)$ to solve

$$\mathcal{C}^* = \arg\max_{\mathcal{C}} EV(\kappa,\mathcal{C},\mathcal{C}'(\mathcal{C},\kappa)), \text{ subject to } E\pi \geq 0. \tag{7}$$

Since the above problem may not always have a solution with $L^* > 0$, we shall define a notion of feasible first round credit limits.

Definition 2. *We say that $L > 0$ is feasible if there exists an interest rate R such that first round lender profits are non-negative when $\mathcal{C}'$ satisfies Lemma 2 and consumers optimally choose borrowing and default given $\mathcal{C} = (L, R)$ and correct beliefs about $\mathcal{C}'$.*

Our main result contrasts equilibrium under prepayment risk to the exclusivity benchmark. To that end, we denote the equilibrium contract and its associated borrowing level b_0 *under exclusivity* by $\mathcal{C}^{\emptyset} = (L^{\emptyset}, R^{\emptyset})$ and $b^{\emptyset}$, respectively. Incidentally, such contract is constrained efficient in the sense that it solves the problem of a benevolent planner who is restricted by the same market incompleteness as lenders are.[10] Furthermore, we define L_{max}

[9] This best response may not be unique. If the agent transfers all her borrowing to $\mathcal{C}'$ but does not fully utilize the line, any line with L' between consumer's borrowing level and $\mathcal{L}_{max} - L$ is also a best response. Since borrowing is given by (4), it does not depend on her beliefs about L' as long as L' is not binding, and all these contracts are equivalent. Thus, it is without loss to focus on this particular best response, which is unique when the balance transfer line is fully utilized.

[10] Specifically, it can be shown that $\mathcal{C}^{\emptyset}$ and $\mathcal{C}' = (0, 0)$ solve $\max_{\mathcal{C},\mathcal{C}'} EV(\kappa,\mathcal{C},\mathcal{C}')$, s.t. $E\pi \geq 0$ and $\pi' \geq 0$.

as the upper bound on L that can be feasibly offered under exclusivity; that is, the highest L that does not trigger default in normal times. Such upper bound is determined by the credit capacity and is thus the solution to a fixed point.[11]

Definition 3. *L_{max} is the credit limit satisfying $L_{max} = \mathcal{L}_{max}(L_{max}, R_{max})$ when $\zeta = 0$, where R_{max} is the lowest interest rate yielding zero expected profits at $L = L_{max}$.*

Finally, we introduce two assumptions which focus our analysis on the set of non-trivial defaultable contracts. Our first assumption warrants in a crude way that consumers always want to default when distressed, and also assumes that contracts exposed to default risk are feasible under exclusivity. A sufficient condition for the former is that default penalties under distress $(1-\theta_x)y$ are lower than the discharged portion of the shock $(1-\phi)\kappa$. This assumption essentially implies that our analytical results pertain to the case when lenders and borrowers do opt for a contract that exposes the lender to a positive risk of default. We refer to such credit lines as *risky* contracts, as opposed to a *risk free* contracts. Hence, our characterization does not cover the choice of the latter over the former, which can be non-trivial in general. In the absence of such assumption, our analysis can be viewed as understanding the effect of prepayment risk on the set of risky contracts that could be possibly sustained in equilibrium.

Assumption 1. $\mathcal{L}_{max}(x, \cdot) = 0$. *There exists a feasible L when $\zeta = 0$.*

Our second assumption warrants that there exists a range of feasible credit limits under which consumers would not be credit constrained under exclusivity. If this was not the case, lenders would always offer the highest credit limit possible to relax borrowing constraints under both exclusivity and non-exclusivity, making the problem trivial. Since full intertemporal smoothing under $\kappa = 0$ is achieved when the agent borrows $B/2$, the following condition assures that there exist non-binding credit limits in our model.

Assumption 2. $B/2 < L_{max}$.

3 Characterization of Equilibrium

The goal in this section is to characterize the equilibrium in our model. Throughout, we use the exclusivity case, $\zeta = 0$, as a reference point. This allows us to assess the impact of

[11] Such a fixed point exists, given that $V^0(0, \cdot)$ is continuous in L and R and bounded while $V^1(0, \cdot)$ is increasing and continuous in L, with $V^0 > V^1$ at $L = 0$.

prepayment risk, which is the central focus of our paper.

The first proposition characterizes the types of contracts that may arise in equilibrium of our model. The first type of contracts, referred to as *rollover* contracts, feature a full balance transfer, yet break even thanks to the existence of positive delay. The second contract type aims at 'strategically' crowding out the credit capacity of the borrower in order to prevent entry all along. Interestingly, such *crowding out* contracts can be sustained even without any delay, as future lenders cannot extended any credit line without tipping the borrower into a 'strategic default zone', i.e., to default regardless the nature of the shock. This, however, requires that the credit capacity is not too large so as to limit initial lender default losses.

Definition 4. *A zero-profit credit line $\mathcal{C}$ is a* ***rollover contract*** *if $b_0 \leq \mathcal{L}_{max}(L, R) - L$. We say that $\mathcal{C}$ is a* ***crowding out*** *contract if $\mathcal{C} = (L_{max}, R_{max})$.*

Now we can state our main analytical result, which implies that equilibrium first round contracts must be either rollover or crowding out when delay frictions are small but positive.

Proposition 1. *There exists $\underline{\zeta} < 1$ such that for all $\zeta \in (\underline{\zeta}, 1)$ the following is true:*

(i) if $L^{\emptyset} \leq \mathcal{L}_{max}(L^{\emptyset}, \frac{R^{\emptyset}}{1-\zeta}) - b^{\emptyset}$ the first round contract is a rollover contract with $(L^*, R^*) = \left(L^{\emptyset}, \frac{R^{\emptyset}}{1-\zeta}\right)$;

(ii) if $L^{\emptyset} \in \left(\mathcal{L}_{max}(L^{\emptyset}, \frac{R^{\emptyset}}{1-\zeta}) - b^{\emptyset}, L_{max}\right)$ the first round contract is either a rollover contract with $L^* = \mathcal{L}_{max}(L^*, R^*) - b_0^* < L^{\emptyset}$ or a crowding out contract;

(iii) if $L^{\emptyset} = L_{max}$ then $(L^*, R^*) = \left(L^{\emptyset}, R^{\emptyset}\right)$.

Moreover, if $\zeta = 1$ then equilibrium involves either no credit provision ($L^* = 0$) or a crowding out contract.

The intuition why delay matters for sustainability of rollover contracts is fairly straightforward. With delay, initial lenders can simply charge a sufficiently high interest rate to fully make up for the revenue lost due to a balance transfer, as their interest rate has no effect on borrowing. This is because borrowing is linear in $(1 - \zeta)R$ under a full transfer by (4). Now, since first period lenders' profit is also proportional to $(1 - \zeta)R$, lenders can always front-load all the revenue that they would otherwise collect in the absence of any prepayment risk. They can do so by simply setting $(1 - \zeta)R$ equal to the interest rate they would charge under exclusivity.[12] In contrast, in the absence of any delay, revenue cannot be front loaded at all. In such a case, the only way defaultable debt can be sustained is by limiting the overall exposure to prepayment risk. The reason why crowding out contracts require credit capacity not to be too large is also easy to see. Crowding out by definition requires that the initial credit limit is close to or at the credit capacity. Since sufficiently high credit limits result in high default losses, this strategy may not be sustainable as there may be no interest rate for the contract to break even.

In addition, Proposition 1 establishes that rollover contracts may actually implement the equilibrium allocation under exclusivity, even though contracts are non-exclusive and future lenders do extend a balance transfer offer. This is the case whenever setting $L = L^{\emptyset}$ leads to a full balance transfer. The intuition is similar to the one given above regarding the sustainability of debt. As long as the balance transfer is full, initial lenders can mimic the contract that would have been optimal under exclusivity. This is accomplished by simply scaling $R^{\emptyset}$ up by a factor $1/(1-\zeta)$. Such adjustment does not affect the borrowing choice of the consumer, as the marginal rate on debt that she faces is equal to $R^{\emptyset}$. While this invariance is particular to the quadratic functional form of intertemporal preferences, it illustrates how, in general, lenders can accommodate future balance transfers by front-loading revenue.[13]

Proposition 1 also shows that when $L^{\emptyset}$ leads to an incomplete balance transfer, then either a rollover contract with an inefficiently low credit limit or a crowding out contract exhibiting the highest credit limit possible (L_{max}) will be offered in equilibrium. Which one is chosen depends on consumer borrowing needs (G and B) and risk aversion (u). As we show in the example below, high borrowing needs and/or high insurance needs typically select crowding out contracts, whereas low borrowing and insurance needs lead to rollover contracts.

Importantly, when balance transfers are incomplete, crowding out contracts always prevail, i.e., $L < L_{max}$ will never be offered in equilibrium (unless it is a rollover contract). The intuition is as follows. Under an incomplete transfer, the marginal interest rate faced by the consumer is R, rather than $(1-\zeta)R$. In this case, as we explain below, lenders have a strong incentive to increase the credit limit all the way to L_{max}. This is because the benefit

[12] When $\mathcal{C}$ is subject to a full transfer, $E\pi = (1-p)(1-\zeta)Rb_0 - pL$, and $b_0 = \frac{B}{2} - \frac{(1-\zeta)R}{8\mu}$.

[13] One could think of quadratic G as a Taylor approximation of standard preferences. In fact, we calibrate it in the quantitative model to approximate a CES utility function.

from crowding out future balance transfers outweighs the additional loss caused by consumers defaulting under distress on a higher credit limit. This allows lenders to *lower* the marginal interest rate R thereby reducing the distortion on intertemporal smoothing when $\kappa = 0$. At the same time, the increase in L raises consumption levels under $\kappa = x$. Consumers like such a change, as it gives them more generous insurance against the distress shock and reduces intertemporal interest rate distortion. The negative relationship between interest rates and credit limits contrasts with the exclusivity benchmark, in which higher default losses must necessarily be offset by higher interest rates. In general, crowding out may be incomplete, but only when defaulting on $L = L_{max}$ would imply *higher* consumption under distress than under no distress, which rule out here by our above assumption that income after defaulting, net of shocks, is higher in normal than in distressed times.

To better illustrate lenders' incentive to crowd out, focus on the case of $\zeta = 1$ and observe that the benefit of increasing $L < \mathcal{L}_{max}(L, R)$ comes from the fact that L' declines at least one-to-one by Lemma 1. As we can see from (5), the increase in revenue caused by raising L by Δ is proportional to the product of the interest rate R and the probability of repayment $(1-p)$. Specifically, since L' goes down by at least Δ and b_0 only depends on R, the increase in revenue is at least $(1-p)R\Delta$. The cost of such increase in L is, on the other hand, proportional to the probability of defaulting p and given by $p\Delta$. Thus, the benefit outweighs the cost whenever $R(1-p) > p$. It turns out that this condition is always true since R must generally be higher than the lowest possible rate at which initial lenders could possibly break even. In the case of frictionless entry, i.e. $(\zeta = 1)$, we know that $(1-p)R(b_0 - L') \geq pL$ for lenders to make non-negative profits, or equivalently $(1-p)R(b_0 - L')/L > p$. Finally, since $b_0 \leq L$, we must have that $(1-p)R > p$. Hence, in such a case lenders can offer a higher L and a lower R as long as $L < \mathcal{L}_{max}(L, R)$.

The above crowding out motive applies as long as delay frictions are not too high. Actually, the bound on the delay friction is a function of pre-existing debt and credit capacity and is typically quite slack – as we show in the proof of Proposition 1, $\underline{\zeta} \leq \frac{B}{2L_{max}}$.

To conclude the discussion of Proposition 1, we present an example that illustrates the properties of equilibrium contracts, which also arise in our life-cycle quantitative model.

Leading example: *utility u is CES with elasticity $\sigma = 2$ and the curvature of inter-temporal aggregator G is $\mu = .6$, which is approximately equivalent to a CES aggregator with elasticity of 2. Income y is normalized to 1 and default costs are 35% of income in case of no distress and 5% of income in the case of distress ($\theta_0 = 0.65, \theta_x = 0.95$). The probability of the distress shock is 5%, and its size is equal to 40% of income ($x = 0.4$). 50% of this shock is defaultable ($\phi = .5$). The entry delay is set equal to $1/4$ of a period ($\zeta = 0.75$).*

In this example, the lower bound on delay frictions above which Proposition 1 applies is $\underline{\zeta} = 0.6$ for a pre-existing debt of 40% of income ($B = 0.4$). For such debt level the equilibrium contract is actually a crowding out contract exhibiting a credit limit $L^* = \mathcal{L}_{max}(R^*, L^*) = 0.332$, which is more than 25% higher than the (constrained efficient) limit under exclusivity $L^{\emptyset} = 0.266$. This is also highlighted by the difference in utilization rates (b_0/L): while 69% of line $\mathcal{C}^{\emptyset}$ is utilized, in equilibrium the utilization rate is only 54%. The oversized credit line leads to an inefficiently high charge-off rate ($pL/(pL + (1-p)b_0)$ of 8.8% in equilibrium versus 7.1% under exclusivity.

The switch between rollover and crowding out contracts given the parameters in the example happens at a debt level of roughly 36.2% of income. That is, agents with $B < 0.362$ go for a rollover contract, while agents with $B > 0.362$ prefer a crowding out contract. For instance, a consumer with $B = 0.25$. will be offered a rollover contract with $L^* = L^{\emptyset} = 0.180$, which is much lower than the credit capacity of 0.339 and thus leaves enough room for a full balance transfer. Such line exhibits an utilization rate of 59%, implying that consumer insurance needs affect contract selection, since they are willing to pay a higher interest rate in order to max out on a bigger credit limit when under distress — after defaulting, (aggregated) consumption under distress is 91% of consumption in normal times, compared to only 73% in the absence of credit markets.

We next proceed with the analysis of two qualitative features of our model that are substantively important for the functioning of the credit market. The first implication pertains to the distribution of interest rates. It shows that increases in the intensity of competition are generally associated with a growing dispersion of interest rates, even if the pricing of default risk is perfectly precise given that there is no ex ante asymmetric information in our model. As already mentioned, growing dispersion of interest rates is a feature of the US data, which has thus far been interpreted as evidence that lenders price default risk more precisely. While this still may be the case, here we show that the fact that balance transfers have become a staple of the credit card market may be a contributing factor to the observed growth in rate dispersion. The second implication is that, while rollover contracts greatly enhance sustainability of defaultable debt despite non-exclusivity of contracts and fierce ex-post repricing, this work-around so to speak comes with strings attached. Specifically, it creates a rollover risk that manifests itself as a hike in interest rates faced by consumers when credit supply is disrupted in some way. As such, it may accelerate deleveraging in response to such shocks or push consumers into default.

Interest Rate Dispersion. Prepayment risk has important implications for the pricing of credit card debt. Specifically, as the next corollary states, defaultable debt exhibits higher interest rates compared to those under exclusivity. This, combined with the low introductory rates associated to balance transfer offers, implies a 'spreading out' of the distribution of interest rates as prepayment options became more prevalent, something that happened during the 90 and 00s as illustrated in Figure 1. Such spreading out follows a particular pattern, which is predicted by our model: the rise of a left tail about rates equal to costs of funds (here normalized to zero) associated to balance transfer offers and a fatter right tail associated to interest rates of contracts exposed to prepayment risk. The latter follows from two different channels. First, revenue front loading associated with rollover contracts requires lenders to set rates proportional to $1/(1-\zeta)$. Hence, for small ζ, rollover contracts exhibit very high rates. Second, high exposure to default losses exhibited by crowding out contracts needs to be priced in the interest rate, leading to $R^* > R^\emptyset$.

Corollary 1. *If delay frictions are small then $R' < R^\emptyset \leq R^*$, with strict inequality whenever $L^\emptyset < L_{max}$.*

Figure 2 conceptually illustrates the distribution of outstanding rates (i.e., the rate on the biggest credit card balance) in our leading example, which is derived for a population in which half of consumers have pre-existing debt $B = 0.25$ and the other half have $B = 0.4$. In the case of balance transfers and rollover contracts the prevalence is weighted proportional to the delay $\zeta = 0.75$. The figure shows that while the distribution of rates is quite concentrated around 8% in the absence of prepayment risk ($\zeta = 0$), it becomes quite dispersed with a left tail at the balance transfer rate and a 'fat' right tail associated to the rate on rollover contracts (35.7%), with the rate on crowding out contracts being in between (9.7%). The introduction of prepayment risk also raises the average interest rate on debt outstanding from 8.25% to 9.3%, and thus distorts the intertemporal margin.

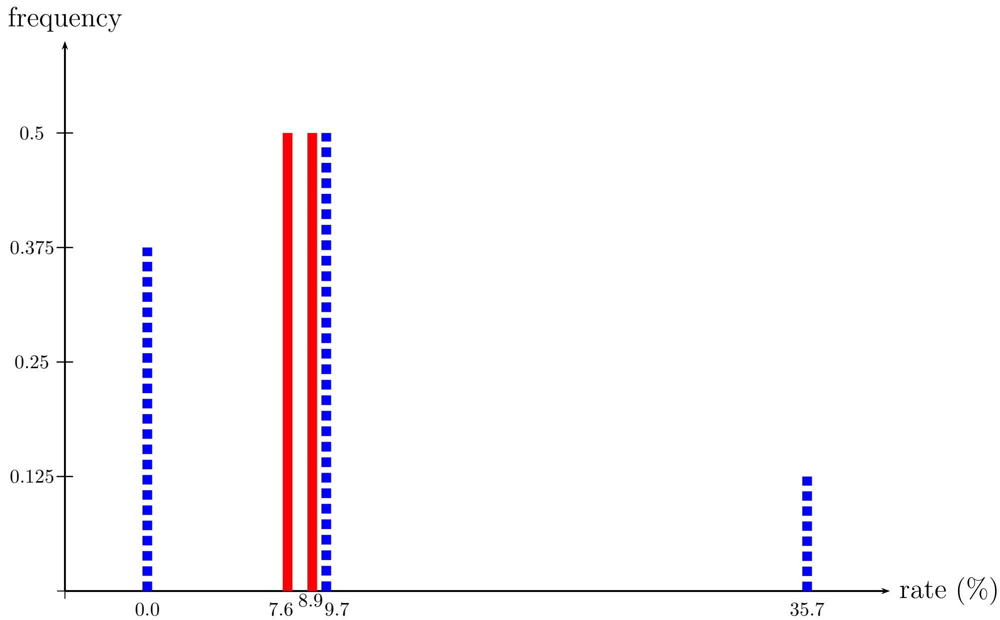

Figure 2: Rate Distribution with (dashed) and without Prepayment Risk (solid).

Market Fragility. Our leading example also highlights two features of equilibrium contracts that generate macroeconomic fragility in the unsecured credit market. First, oversized credit limits associated with crowding out contracts lead to an excessive debt discharge by distressed consumers. Consequently, a higher occurrence of distress due to macroeconomic shocks may lead to inefficiently high charge-offs. In addition, if the market is hit by an unanticipated credit shock that reduces the availability of repricing opportunities, as seems to have been the case during the 2007-09 financial crisis, the high rates exhibited by rollover contracts could lead to a rapid deleveraging by consumers facing a major hike in marginal interest rates on debt. In practice, it may also lead to higher default rates although this possibility is absent in our model due to the binary nature of the shock. This second mechanism is somewhat analogous to the case of sub-prime mortgages and the wave of defaults that resulted from the collapse of housing prices and the ensuing inability of borrowers to refinance those mortgages.

To illustrate the key mechanism behind prepayment induced fragility, consider again our leading example assuming that half of consumers hold rollover contracts and half crowding out contracts. The charge-off rate in such a case is 8.6% and the outstanding credit card debt, net of charge-offs, is 13.6% of income. Now, imagine the following scenario: after the first round contracts have been accepted, half of consumers with rollover contracts learn that they will not get a balance transfer offer (*credit shock*); and, simultaneously, the probability of the distress shock jumps from 5 to 7% (*macroeconomic shock*). Clearly, the macroeconomic shock by itself raises the charge-off rate to 11.9%. However, since the credit shock also leads to a reduction of borrowing levels for those with rollover contracts, it results in both substantial deleveraging and a further increase in the charge-off rate due to lower outstanding credit card balances. In particular, in our leading example consumers would reduce their balances from 10.6% to 5.1% of income, causing average outstanding debt to drop to 12.0% of income. This implies a deleveraging of more than 12% and leads to a final charge-off rate of 13.0%. In contrast, deleveraging under exclusivity would be just 2% and the charge-off rate would increase by just 2.9 percentage points (from 7.5% to 10.4%).

We finish our discussion of results by mentioning that changes in the regulatory environment affect the credit capacity and thus the nature of contracts in equilibrium. Hence, one needs to be careful about imposing regulations that do not account for the potential effects of prepayment risk, but affect the credit capacity of borrowers. Such policies can have unintended consequences according to our model. A relevant example is the means testing regulation imposed by the bankruptcy law reform of 2005. According to our model, the implied increase in default penalties can be beneficial if θ_0 was inefficiently low so that it restricted borrowing and thus consumption smoothing. However, it can also exacerbate the detrimental effects of balance transfers and contribute to market fragility. None of these predictions arise in standard models of unsecured credit and thus more analysis is needed to better understand their quantitative impact.

4 Quantitative Analysis

Here we extend our setup to a life-cycle model that we parameterize by matching moments in the US data. The goal of this exercise is twofold. First, we want to demonstrate that under plausible conditions the model can account for the level of balance transfers and interest rate dispersion seen in the data. Second, using the calibrated model, we explore the effect of repricing under non-exclusivity on market fragility in light of the deleveraging observed during the recent financial crisis. Specifically, we model the crisis by increasing the frequency of expense shocks in the population, and assume that repricing opportunities unexpectedly worsen for consumers in consistency with the decline in credit card solicitations (in 60% of cases the repriced contract does not arrive). By restricting the credit supply shock to only affect balance transfer opportunities rather than overall access to credit, our aim is to quantify the potential impact of the repricing channel alone.

Our quantitative results show that, first, a two year disruption of repricing opportunities can result in a hike in interest rates paid on credit cards that matches the one observed in the aggregate data. Second, we show that our model comes close to fully matching the extent of deleveraging observed during the first two years of the recent financial crisis. In contrast, we show that none of these features can be matched by a counterfactual model that assumes exclusivity of credit card contracts and features no prepayment risk. We see this experiment as an important stepping stone in assessing the impact of the understudied combination of non-exclusive contracts and intense ex-post competition through balance transfer offers.

4.1 Multi-period Life-cycle Model

In our multi-period model consumers live for 27 periods. The first 22 periods are working age periods, during which consumers are subject to stochastic income y. Income follows a Markov process, and in retirement periods y is a deterministic function of realized income in the period right before retirement $y_{22+t} = f(y_{22}),\ t = 1, 5$. Relative to our earlier setup, B is now endogenous.

Within each long period of the full model we embed our two-period model with some changes which connect the periods. Specifically, we allow the consumer to carry over debt into the future. Formally, the second period budget constraint is now replaced by:

$$c' = Y + B' - b - \rho(b, \mathcal{C}, \mathcal{C}'),$$

where B' is the debt that the consumer carries over into the following long period.

The value functions that describe consumers involve B and y as state variables. Specifically, we assume that a consumer who does not have a default flag on record at the interim stage (after she sees κ), and decides not to default in the current period, solves the following dynamic program:

$$V_t^0(B, y) = \max_{c_1, c_2, B'} \{u(G(c_1, c_2)) + E\beta V_{t+1}^0(B', y')\}$$

where c_1, c_2 satisfy the budget constraints implied by the equilibrium contracts $\mathcal{C}, \mathcal{C}'(\mathcal{C}, \kappa)$ in interim state (B, y, κ) and V_{t+1}^0 denotes the continuation value of an agent with no default on record.

In the above problem, we assume that consumers choose c_1, c_2 within each period, and face an almost analogous budget constraint to the two-period model. The value function V_t^1 for a consumer who chooses to default can be defined analogously, and is omitted. The ex-ante value function V_t is given by equation (1).

The cost of defaulting additionally involves a one period exclusion to autarky. Specifically, when in autarky, the consumer can save but cannot borrow. Furthermore, if the consumer experiences another distress shock, she is only able to rollover a fraction ϕ of the shock at a penalty interest rate $\bar{r}$ to the next period. The remainder of the shock, $(1 - \phi)\kappa$ must be absorbed by current consumption. In the next period the default flag is removed, and the consumer starts fresh with $\delta = 0$ and $B = (1 + \bar{r})\phi\kappa$. This additional feature is meant to assure that consumers cannot default serially, which does not happen in the data.

4.2 Parameterization

We require our model to be consistent with key moments in the US data for year 2004. We have chosen 2004 as the baseline year because of a major bankruptcy reform in 2005. Since we calibrate the steady state of the model, we chose to avoid statistics that may be contaminated by the crisis experienced during 2007-09.

The model period is two years long. This implies that each subperiod is one year long. Since default statistics reported in the data are annual, while default in our model can only happen once per two years, measurement must be appropriately adjusted. To this end, all model implied statistics that are flow variables and pertain to default, such as debt charged-off by banks or the rate of default, are divided by two. The only exception from this rule are balance transfers. This is because balance transfers pertain to dynamics across periods, whereas in our model consumers only have one such opportunity before the contract expires. Hence, we chose to compare the raw statistic from the model to the data.

We next describe our data targets and selection of parameter values.

Parameters selected arbitrarily. Our model is complex and there are many parameters that we need to calibrate. To this end, we start from a set of parameters for which we lack a good data anchor, and we choose their values arbitrarily. These are: entry delay ζ and penalty interest rate $\overline{r}$. In the benchmark model, we set $\zeta = 0.75$. That is, we assume entry is on average delayed by six months. We also consider the case of exclusivity ($\zeta = 0$). This counterfactual economy is parameterized analogously. Finally, the penalty interest rate is set equal to 35% per annum. We also assume that u is CES with risk aversion coefficient $\sigma = 2$.

Parameters independently selected to match the data. We calibrate the income process y and the distress shock κ using income and distress data reported by Livshits, MacGee and Tertilt (2010). Specifically, starting from the usual annual AR(1) process for income taken from Livshits, MacGee and Tertilt (2010), we assume that any income drop above 25% is attributable to the distress shock. We then convert the residual to obtain a biannual Markov process using the Tauchen method, which gives us the 6x6 transition matrix P and values for the associated income grid points. We start our simulation from the ergodic distribution of this Markov process.

As mentioned, the distress shock κ absorbs income shocks of 25% or more. In addition, we augment this shock by including three major lifetime expense shocks that were singled out as important by Livshits, MacGee and Tertilt (2010): medical bills, the cost of an unwanted pregnancy, and divorce costs. We use their estimated values, although appropriately adjusted to obtain a single biannual distress shock. The procedure gives us $x = .4$ (40% of median annual household income), and shock frequency of 7.7% on a biannual basis ($p = 0.077$).

We consider here medical bills to be the only shock that can be directly defaulted on, and consequently set $\phi = .24$. This approach departs from the usual practice of treating the distress shock as almost fully defaultable. In contrast to the literature, in our model only a small fraction of the shock is actually accounted for by medical bills, which results in a largely 'non-defaultable' distress shock. It is known that such, arguably more realistic, approach makes it very difficult for this class of models to account for the relatively high frequency of default in the data. To account for default related statistics, we follow the approach proposed by Drozd and Serrano-Padial (2013), who argue that, when the predominance of informal default over formal bankruptcy filings in the data is taken into account, the pecuniary cost of default is endogenously state contingent. Such approach can help account for high levels of default in the presence of high levels of gross unsecured debt. Accordingly, we exogenously set the state-contingent spread on pecuniary default penalties so as to match observed gross debt and default rates. Our calibrated model is consistent with most aspects of the data as far as debt and default is concerned.

Parameters jointly selected to match the data. Having set the parameters governing repricing, distress and income shocks, we next calibrate the remaining parameters to simultaneously hit several data targets. The targets include the key characteristics of the US credit card market in 2004, such as charge-off rate, debt-to-income ratio, balance transfer rate, and debt defaulted on to income ratio per defaulting household. The calibration is joint because most parameters affect several targets at the same time. The parameter values that we choose simultaneously are: $r, \beta, \theta_0, \theta_1$. The associated targets are detailed in Table 1, along with their calibrated values in the model. Since we only have data for the fraction of balances transferred in 2002 (17%), we use the average annual growth rate on balance transfers reported by Evans and Schmalensee (2005) in order to obtain the targeted value for 2004, which is equal to 20%.

Table 1: Data moments characterizing the US credit card market

Moment	Data target	Model	
		Non-Exclusivity ($\zeta = .75$)	Exclusivity ($\zeta = 0$)
A. Targeted moments			
CC debt to disposable income	9.2%	9.2%	9.2%
Net charge-off rate	4.7%	4.65%	4.7%
Debt discharged to income per defaulting hh[a]	93%	93%	60%
Average interest rate on cc debt[b]	10.95%	10.66%	10.93%
Balance transfer rate per annum[c]	20%	20%	na
B. Endogenous moments			
Annual frequency of default per 1000 persons[d]	8.6/1000	9/1000	9/1000
Interest rate dispersion (coef. variation)[e]	64% in 2004, $\approx$ 15% in 1990	56%	15%

Data values pertain to US data for year 2004, unless otherwise noted.
[a] Target consistent with data for formal bankrupts in Sullivan, Westbrook and Warren (2001).
[b] Debt weighted interest rate on revolving accounts assessing interest rate. Data from Federal Reserve Board.
[c] Data from Evans and Schmalensee (2005) for 2002, extrapolated to 2004 based on approximate historic annual rate of growth.
[d] Formal bankruptcy filings; includes Chapter 7 and chapter 13, per 1000 persons 20 years old or older. Data from American Bankruptcy Institute. May include some business filings, especially chapter 13. On the other hand, does not include informal bankruptcies, which our model also captures by targeting the net charge-off rate.
[e] Authors' calculations using SCF data on interest rates on the main/most recent credit line of revolvers. Coefficient of variation in the data is not weighted by debt, in the model it is weighted. Results were similar when we did not weight by debt, and so we decided to report the weighted statistic. The number for 1990 is extrapolated using regression analysis based on the available time series 1995-2004.

4.3 Sample Simulation

Our calibration procedure directly aims at delivering a balance transfer rate of about 20% per annum, among other targets. Consequently, we only test the model's ability to generate substantial repricing as an endogenous equilibrium outcome. We nevertheless think of it as a remarkable result, as it is far from obvious that a mere presence of a balance transfer option will lead to this option being used so frequently in equilibrium. This is possible in our model due to the use of rollover contracts.

To illustrate the workings of our calibrated model, Figure 3 illustrates a sample simulation of an agent in our economy. The top panel shows the evolution of income of this agent, with "x" denoting distress shocks. This is a relatively low income agent, who experiences one distress shock during her life. This shock triggers a default early on. The bottom panel plots the life-cycle of credit for this agent: specifically, the credit limit L offered in equilibrium, the counterfactual credit limit $L^{\emptyset}$ lenders would offer under exclusivity, and the size of the balance transfer offer L'. The label "-" identifies cases when the underlying contract is exposed to default risk, i.e., the agent would default if hit by a shock. As we can see, balance transfers (i.e., $L' > 0$) are quite prevalent, with about 22% of agent's credit lines exposed to default being rollover contracts. Incidentally, crowding out contracts with excessive credit limits relative to exclusivity are the predominant form of credit for this household (61%). In the overall population of consumers, crowding out contracts are less frequently used. Specifically, among all contracts exposed to default risk, 33% are rollover contracts and also 33% are crowding out contracts, exhibiting credit limits at least 3% higher than $L^{\emptyset}$.[14]

4.4 Cross-Sectional Implications

Table 1 reports the key endogenous implications of our model. These pertain to interest rate dispersion and frequency of default. Following our theoretical analysis, we compare two setups: non-exclusivity with $\zeta = .75$ and exclusivity or $\zeta = 0$. The latter regime is parameterized analogously. Our model matches well the dispersion of interest rates for year 2004. Moreover, the extrapolated value of interest rate dispersion for year 1990 comes remarkably

[14]The remaining 34% of contracts exposed to default risk are crowding out contracts with $L = L_{max} \approx L^{\emptyset}$.

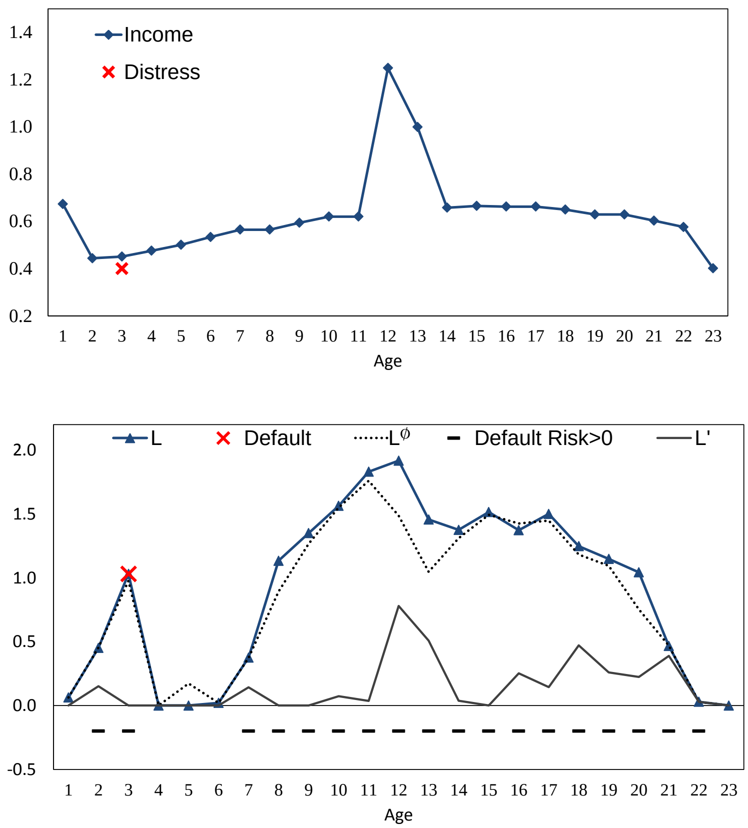

Figure 3: A Sample Household.

close to the dispersion implied by the exclusivity case. Given that balance transfers were nonexistent back then (see Evans and Schmalensee (2005)), the exclusivity regime appears to be a good description of the data in the early 90s. Accordingly, the comparison across regimes shows that the rise in interest rate dispersion in the US credit card market may be merely a by-product of intense competition through ex-post repricing. Thus far it has been attributed to improvements in the pricing of default risk (e.g. Livshits, MacGee and Tertilt, 2011).

Our model also accounts for total bankruptcy filings in the US. However, an important caveat applies here. Namely, consumers often default informally in the data, which means that they do not pay their debt yet do not formally file in court. Consequently, as a measure of the overall default rate, this data target is likely understating the facts. Nevertheless, we are quite confident that we could match a higher level of default, had we lowered our target for the average debt defaulted on per bankrupt. Existing studies suggest that informal bankrupts default on smaller amounts. At this point more precise data is required to get a good handle on the frequency and nature of informal bankruptcy in the US.

4.5 Default and deleveraging During the 2007-09 Crisis

One of the key implications of our theory is that balance transfers distort credit contracts offered in equilibrium. In particular, balance transfers in equilibrium introduce *rollover risk*, since both are associated with rollover contracts designed to secure interest revenue for incumbent lenders. Such rollover risk materializes if entry of future lenders is disrupted by a credit supply shock. This is because in the case of disrupted entry (repricing opportunities), the borrower simply faces a hike in the interest rate charged on debt. Such hike can either increase the default rate or accelerate repayment of existing debt. In our model it does the latter, since we only have two states, distress and no distress. If such a credit crunch is aggregate in nature, it may lead to a counter-cyclical deleveraging in the economy, despite the fact that maturity of credit card contracts is relatively long (5-6 years).

We next explore the potential of our model to explain the key changes in the credit card market observed during the recent financial crisis. In particular, we ask whether it can replicate the sharp reduction in credit card debt to income and the increase in interest rates during the crisis. We do so in consistency with the dramatic decline in credit card solicitations, which in our model we associate with a decline in repricing opportunities.

To set up this quantitative exercise, we target a change in three moments in the data between 2007 and 2010 (peak to trough change) by introducing two unanticipated changes: an increase in the frequency of distress shock p and a positive probability $(1-\xi)$ of disrupted entry of second round lenders in the middle of the period (i.e., the second round happens with probability ξ). This first parameter is chosen to match the change in the charge-off rate; the second parameter is reduced from 1 to .42 to match the drop in credit card solicitations of 58% in the data.

Figure 4 summarizes the results of the experiment. We compare two cases. In the first case repricing opportunities are disrupted as described above. In the second case the market operates under exclusivity, and the model is calibrated to the same data targets. The only parameter that is different across these two cases is the discount factor β and the risk free interest rate r, as the exclusivity case generally leads to more debt-to-income in equilibrium due to cheaper credit (recall that $C^{\emptyset}$ exhibits lower marginal rates than crowding out contracts).

As Figure 4 shows, the model under exclusivity falls short of rationalizing the observed deleveraging and does not capture the increase in interest rates. This is because the credit shock only affects the availability of balance transfer offers, without having any impact on first round contracts. In contrast, the benchmark model fits the data almost perfectly. Specifically, debt-to-income falls from 9.2% to 8.4% in the model, compared to the drop from 9.2% to 8.3% in the data, whereas in the case of exclusivity the drop is only to 8.7%, as it is solely driven by the spike in charge-offs caused by the macroeconomic shock. The ability of our model to match the observed deleveraging is due to the added impact of some agents being 'stuck' with rollover contracts and thus borrowing less than expected. This channel represents about 16% of the overall deleveraging in the model, the rest being driven by debt charged off due to higher default rates. Most importantly, the entry disruption makes the average interest rate on existing debt jump by 2.4 percentage points, almost exactly matching the observed change in the data (2.5 pp). For obvious reasons, this is not the case under exclusivity.

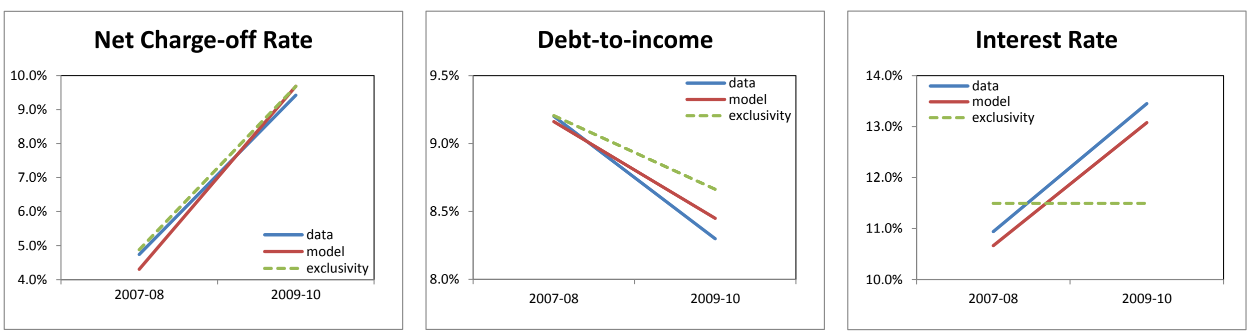

Figure 4: The Effect of Credit Shocks.

The effect of disrupting repricing is also fairly asymmetric in our model. The 2.5 p.p. increase in the average interest rate stems from an interest rate hike faced by only about 12% of revolving contracts. Such rate hike is on average about 16 p.p., leading these agents to borrow 8% less on average.

These results highlight the importance of the rollover risk created by balance transfers and demonstrate that the induced market fragility can be quantitatively relevant given the observed level of balance transfers in the US data. It is important to emphasize that the disruption of credit supply here merely affects the ability of consumers to reprice credit card debt within just one period. In particular, it does not reduce access credit during the crisis period and in the period thereafter. We consider this as the most realistic way of introducing credit supply shocks, given the relatively long maturity of credit card contracts in the data, and the fact that banks after 2009 were no longer allowed to force repayment nor increase interest rates on pre-existing debt. In such context, a change in households' time preferences or the lack of continuation contracts beyond expiration of existing ones seem the only plausible remaining alternatives to the short-term disruption of debt repricing explored here.

5 Conclusions

We have developed here a formal model of prepayment risk in the context of the US credit card market. We have demonstrated that the model can match both the level of balance transfers seen in the data and the dispersion in interest rates, while being consistent with other statistics characterizing this market. We have used the model to better understand the events of the most recent financial crisis in the credit card market. In particular, we assessed the potential of prepayment risk to account for deleveraging in the credit card market and for the observed hike in credit card interest rates. Our results suggest that the unexpected lack of repricing could have importantly contributed to the observed deleveraging. In terms of policy recommendations, our analysis suggests that intense ex-post repricing under non-exclusive contracts may be detrimental to consumers, as it not only leads to distortions in intertemporal smoothing but also to macroeconomic fragility in response to short-lived aggregate disruptions in the credit supply.

Appendix

A1. Proof of Proposition 1

Part (i) immediately follows from the argument in the text. Part (iii) is obvious: if the credit capacity constraints lenders under exclusivity then first round lenders offer the same contract when $\zeta > 0$.

We prove Part (ii) by laying out the proof argument using a series of Lemmas that are then proved below. The proof goes as follows.

First, focus on the aggregated consumption combinations (c_0, c_x) that involve positive credit provision ($L > 0$), where $c_\kappa = G(c_{1,\kappa}, c_{2,\kappa})$ are candidates for equilibrium aggregate consumption under exclusivity. We call such set of candidates the exclusivity consumption frontier (ECF). Since the agent defaults under distress by Assumption 1, c_x is increasing in L and independent of R, so the ECF is obtained by increasing the credit limit L and adjusting the interest rate so that lenders make zero profits. The solution to (7) when $\zeta = 0$ must then maximize consumers' expected utility, which is represented by convex indifference curves on the space of (c_0, c_x), with slope $-\frac{1-p}{p}\frac{u'(c_0)}{u'(c_x)}$. Thus, if we pin down the properties of the ECF we should be able to identify the allocation under exclusivity and the contract implementing it. The first step towards characterizing the ECF is to show the relationship between credit limits and interest rates on the frontier. Let $\Delta R/\Delta L$ refer to the change, after an increase of ΔL in the credit limit, in the lowest interest rate lenders can feasibly charge at L.

Lemma 3. $\Delta R/\Delta L > 0$ *along the ECF.*

This positive relationship between L and R implies that, as c_x is raised by increasing L, the marginal rate faced by consumers goes up and so does the distortion of intertemporal smoothing under $\kappa = 0$. This leads to an ECF with the following properties. Let c_{Lmax} denote the highest feasible aggregated consumption under distress, i.e., the one associated with contract (R_{max}, L_{max}).

Lemma 4. *The ECF is continuous and concave. Furthermore, there exists* $\underline{c}_L < c_{Lmax}$ *such that the ECF is flatter than* $-(1-p)/p$ *at all points with* $c_L \in [\underline{c}_L, c_{Lmax}]$.

Accordingly, since indifferent curves are convex with slope $-(1-p)/p$ on the diagonal ($c_0 = c_x$), the optimal allocation will exhibit $c_x < c_{Lmax}$, i.e., $L^\emptyset < L_{max}$, except in the case that indifference curves are flatter than the ECF at c_{Lmax}, in which case a corner solution arises and $L^\emptyset = L_{max}$.

We then turn to the equilibrium allocation and proceed in a similar fashion. First, we identify the set of (c_0, c_x) that are candidates for equilibrium, which we call the profit feasible consumption frontier (PFCF) and show that, when ζ is high enough, interest rates and credit limits have a negative relationship at the PFCF.

Lemma 5. *If $\zeta \geq \underline{\zeta}$ then $\Delta R/\Delta L < 0$ for all non-binding L satisfying $L > \mathcal{L}_{max} - b_0$.*

Lemma 5 has the following implications for the slope of the PFCF:

Lemma 6. *If $\zeta \geq \frac{1}{2}B/L_{\max}$ then the slope of the PFCF is steeper than $-(1-p)/p$ at all points involving incomplete balance transfers.*

This result then implies that, when equilibrium is implemented by a contract subject to incomplete transfers, it must exhibit a credit limit $L^* = L_{max}$. The reason is that consumption combinations on the frontier exhibit $c_x \leq c_0$. But then indifference curves at those points are flatter than $-(1-p)/p$, leading to a corner solution in which $c_x = c_{Lmax}$.

To see why $c_x \leq c_0$, notice that the highest c_x and the lowest c_H are associated to $L = L_{max}$. But at that point the agent is indifferent between defaulting or not in normal times. That is, c_0 equals aggregated consumption contingent on default when $\kappa = 0$. Since L_{max} is not binding by Assumption 2, this means that, in the event of default, the agent fully smooths consumption intertemporally, yielding

$$c_0 = (1+\theta_0)y + L_{max} - B \geq (1+\theta_x)y - \phi x + L_{max} - B = c_x,$$

where the last inequality comes from our assumption that $\theta_0 y \geq \theta_x y - \phi x$.

To finish the proof of part (ii) we argue that if equilibrium is implemented by a rollover contract with $L^* < L^\emptyset$ then, by the concavity of the ECF, it must exhibit the highest credit limit among contracts subject to a full transfer. This follows directly from the fact that the exclusivity problem is isomorphic to the lenders' restricted problem of choosing among rollover contracts and so the portion of the PFCF associated to rollover contracts coincides with the segment of the ECF exhibiting the same first round credit limits.

Finally, notice that when $\zeta = 1$ rollover contracts are not sustainable and thus equilibrium must exhibit either no credit provision or an incompletely poached contract, which must be a crowding out contract by the argument above. This completes the proof.

A2. Remaining Proofs

Proof of Lemma 1. We prove the result for $\kappa = 0$. The proof for $\kappa = x$ is analogous and therefore omitted. By definition, $\mathcal{L}_{max}(0; L, R, R')$ is given by the lowest aggregate credit limit $L + L^*$ such that $V^0(0, (L, R), (L', R')) > V^1(0, (L, R), (L', R'))$ for all $L' > L^*$.

To show that $\mathcal{L}_{max}(0; L, R, R')$ is bounded from above, note that for all $L \geq 0$ if $L + L' > (1-\theta_0)y$ we must have $V^0(0, (L, R), (L', R')) < V^1(0, (L, R), (L', R'))$ as long as aggregated consumption is increasing in $c_{1,\kappa}$ and $c_{2,\kappa}$. This is because second period income is higher after maxing out and defaulting on $L + L'$ while first period income and borrowing constraints are the same under repayment and default. This implies that the agent can enjoy higher consumption on both periods under default.

We next show that $\mathcal{L}_{max}(0; L, R, R')$ is decreasing in L as long as $b_0 < L$. We can express aggregated consumption under repayment and default respectively as

$$\begin{aligned} V^0(0,&(L,R),(L',R')) = \\ &2y - B - (1-\zeta)Rb_0 - \zeta R\max\{b_0 - L', 0\} - \zeta R' \min\{b_0, L'\} - \mu(B - 2b_0)^2, \end{aligned} \tag{A1}$$

and

$$V^1(0,(L,R),(L',R')) = (1+\theta_0)y - B + L + L' - \mu(\max\{0, B - (1-\theta_0)y + L' - L\})^2, \tag{A2}$$

where the last term in (A2) comes from the fact that, when L' is high enough, L will be binding under default. In addition, borrowing b_0 in (A1) is given by

$$b_0 = \begin{cases} \max\left\{0, \min\left\{L, \frac{B}{2} - \frac{R}{8\mu}\right\}\right\} & L' \leq \min\left\{L, \frac{B}{2} - \frac{R}{8\mu}\right\} \\ \max\{0, L'\} & \min\left\{L, \frac{B}{2} - \frac{R}{8\mu}\right\} < L' \leq \min\left\{L, \frac{B}{2} - \frac{(1-\zeta)R + \zeta R'}{8\mu}\right\} \\ \max\left\{0, \min\left\{L, \frac{B}{2} - \frac{(1-\zeta)R+\zeta R'}{8\mu}\right\}\right\} & L' > \min\left\{L, \frac{B}{2} - \frac{(1-\zeta)R+\zeta R'}{8\mu}\right\}, \end{cases} \tag{A3}$$

and reflects the fact that the agent faces two constraints: the borrowing constraint L and also, the balance transfer constraint L'. It is easy to see that (A1) does not change after an increase in L given by ΔL when $b_0 < L$ whereas (A2) increases by at least ΔL. Thus, since a reduction in L' equal to ΔL exactly offsets the effect on V^1 of the increase in L, but such reduction in L' lowers V^0, $\mathcal{L}_{max}(0; L, R, R')$ must have gone down after the increase in L. That is, $\mathcal{L}_{max}(0; L, R, R')$ is decreasing in L as long as $b_0 < L$. □

Proof of Lemma 3. First note that, along the ECF, lenders charge the lowest feasible interest rate, i.e., R is the smallest solution to

$$(1-p)Rb_0 = pL.$$

In addition, borrowing constraints will never be binding along the ECF when $\kappa = 0$ unless $L^{\emptyset} = L_{max}$ is binding, which is ruled out by Assumption 2. This is because, whenever L is binding, the interest rate that satisfies the zero profit condition $(1-p)RL = pL$, is independent of L and given by the full utilization rate $R = p/(1-p)$. Hence, they can increase L without increasing R, thereby relaxing the borrowing constraint on the repayment path while increasing consumption on the default path. This leads to higher aggregated consumption under both distress and non-distress.

Thus, for non-binding $L < L_{max}$, interest revenue is given by $R\left(\frac{B}{2} - \frac{R}{8\mu}\right)$, which is a continuous, concave function of R, initially increasing (and equal to zero) at $R = 0$ and reaching a maximum at $R = 2\mu B$. Thus, at each L lenders would choose the interest rate in $(0, 2\mu B]$ satisfying the above zero profit condition. But then, since Rb_0 is strictly increasing for all $R \in [0, 2\mu B)$ any increase of L on the ECF must be accompanied by an increase in R. $\square$

Proof of Lemma 4. As argued in the proof of Lemma 3, credit limits are not binding on the ECF when $\kappa = 0$. In addition, Assumption 2 also guarantees that there is a range of credit limits that are also non-binding under $\kappa = x$. To see this is the case, recall that $\theta_0 y \geq \theta_x y - \phi x$. Since L_{max} is bounded above by $(1-\theta_0)y$ we then must have that

$$y - L_{max} \geq \theta_0 y \geq \theta_x y - \phi x.$$

Now, since $B < 2L_{max}$ by Assumption 2, adding $2L_{max} - B$ to the LHS of the above expression yields $y - B + L_{max} > \theta_x y - \phi x$, i.e., consumers under distress are not credit constrained at credit limits close to L_{max}.

We also know that, since profits are zero for any contract associated to a point on the ECF, interest payments satisfy $Rb_0 = \frac{p}{1-p}L$. Using this expression and substituting $b_0 = B/2 - R/(8\mu)$ into (A1) we can write aggregated consumption when $\kappa = 0$ as

$$c_0 = V^0((L, R), (0, 0)) = 2y - B - \frac{p}{1-p}L - \frac{R^2}{16\mu}.$$

In addition, aggregated consumption when $\kappa = x$ is given by

$$c_x = \begin{cases} (1+\theta_x)y - B - \phi\kappa + L - \mu(B - L - \phi\kappa - (1-\theta_x)y)^2 & L < B - \phi\kappa - (1-\theta_x)y \\ (1+\theta_x)y - B - \phi\kappa + L & L \geq B - \phi\kappa - (1-\theta_x)y, \end{cases}$$

where the first case reflects the fact that L may be binding. Given these expressions, the continuity of the ECF is straightforward. Also, it is easy to see that c_x increases one-to-one with L when L is not binding while, given Lemma 3, c_0 decreases by more than $p/(1-p)$ when L goes up by one. Hence, the ECF must be flatter than $-(1-p)/p$ at all points associated with $L \in [B - \phi\kappa - (1-\theta_x)y, L_{max}]$. But since L_{max} is not binding and c_x is increasing in L, this interval of credit limits maps onto an interval $[\underline{c}_x, c_{Lmax}]$ of consumption under distress for some $\underline{c}_x < c_{Lmax}$.

Finally, to show concavity, it suffices to show that c_0 and c_x are concave in L, given that c_0 is strictly decreasing and c_x is strictly increasing in L. If we differentiate twice the above expression of c_0 w.r.t. L we get that

$$\frac{d^2 c_0}{dL^2} = -\frac{dR}{dL}\frac{1}{8\mu} - \frac{d^2 R}{dL^2}\frac{R}{8\mu}.$$

Since $\frac{dR}{dL} > 0$ by Lemma 3, this expression is negative whenever $\frac{d^2R}{dL^2} > 0$. By implicitly differentiating the zero profit condition we find that

$$\frac{d^2 R}{dL^2} = \frac{p}{1-p}\frac{1}{4\mu(B/2 - R/(4\mu))^2}\frac{dR}{dL},$$

which is always positive for all $R < 2\mu B$. Similarly, c_0 is concave in L:

$$\frac{d^2 c_0}{dL^2} = \begin{cases} -2\mu & L < B - \phi\kappa - (1-\theta_x)y \\ 0 & L \geq B - \phi\kappa - (1-\theta_x)y. \end{cases}$$

□

Proof of Lemma 5. The proof logic is to show that, when the condition in the lemma is satisfied, first round lender profits go up when L is increased while keeping fixed the interest rate. Such rate is the lowest R satisfying the zero profit condition $E\pi(0,(L,R),(0,\mathcal{L}_{max}(L,R) - L)) = 0$, given that lenders must earn zero expected profits at all points on the PFCF. Accordingly, if profits go up with L initial lenders can increase L and lower the interest rate while earning zero profits.

Expected profits are given by

$$E\pi(0,(L,R),(0,\mathcal{L}_{max}(L,R)-L)) = (1-p)R\left[\zeta(b_0 - (\mathcal{L}_{max}(L,R)-L)) + (1-\zeta)b_0\right] - pL. \tag{A4}$$

Since b_0 does not depend on L when borrowing constraints are non-binding, the change in profits after a change of ΔL is given by

$$\frac{\Delta E\pi(0,(L,R),(0,\mathcal{L}_{max}(L,R)-L))}{\Delta L} = (1-p)\zeta R\left(1 - \frac{\Delta\mathcal{L}_{max}(L,R)}{\Delta L}\right) - p.$$

By Lemma 1 we know that $\frac{\Delta \mathcal{L}_{max}(L,R)}{\Delta L} < 0$. Accordingly, a sufficient condition for profits to go up is

$$\zeta R > \frac{p}{1-p}. \tag{A5}$$

From the above zero profit condition we can express ζR as

$$\zeta R = \frac{p}{1-p} \frac{\zeta L}{[\zeta(b_0 - (\mathcal{L}_{max}(L,R) - L)) + (1-\zeta)b_0]}.$$

Accordingly, (A5) holds as long as

$$\zeta L > \zeta(b_0 - (\mathcal{L}_{max}(L,R) - L)) + (1-\zeta)b_0,$$

which yields

$$\zeta > \frac{b_0}{\mathcal{L}_{max}(L,R)}.$$

But this condition must hold as long as $\zeta \geq \frac{B}{2}\frac{1}{L_{max}}$, given that $b_0 < B/2$ for all $R > 0$ and $\mathcal{L}_{max}(L,R) \geq L_{max}$ by Lemma 1. $\square$

Proof of Lemma 6. Since $L' = 0$ when $\kappa = x$, c_x follows the same expression as under exclusivity and thus increases with L by at least one-to-one. Thus, to show that the slope of the PFCF is steeper we just need to show that c_0 decreases by less than $-\Delta L p/(1-p)$ after an increase in L equal to ΔL.

Consider first the case of L non-binding when $\kappa = 0$. Since lenders earn zero profits on the PFCF, we can write c_0 as

$$c_0 = 2y - B - \frac{p}{1-p}L - \frac{R^2}{16\mu}.$$

By Lemma 5 we know that $\Delta R/\Delta L < 0$ and thus the last term goes down as L goes up, implying that $\Delta c_0/\Delta L > -pL/(1-p)$ on the PFCF.

Now consider the case of binding L. Then, we must have $B > 2b_0 = 2L$. In this case,

$$c_0 = 2y - B - \frac{p}{1-p}L - \mu(B-2L)^2.$$

From this expression, it is clear that $\Delta c_0/\Delta L > -pL/(1-p)$ also when L is binding. $\square$

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