Coordination in Global Games with Heterogeneous Agents

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Abstract

This paper studies equilibrium selection in large coordination games played by heterogeneous agents, such as models of bank runs, currency attacks or technology adoption. Player payoffs are affected by the average action and the player's type, as well as a global parameter reflecting economic fundamentals. I introduce the notion of *ex ante risk dominance* and show that it coincides with the global games selection in games with payoffs that are separable in average action and type. Ex ante risk dominance provides an economic interpretation behind the global games selection in terms of maximizing ex ante expected payoffs under pessimistic beliefs. I characterize the ex ante risk dominant equilibrium, pinning down the presence of tipping points in terms of economic fundamentals. I also show that payoff separability is needed for the global games selection to be uniform, that is, to be robust to changes in the distribution of signal noise.

Keywords: global games, heterogeneity, risk dominance, uniform selection

1 Introduction

Coordination problems are central to the study of many economic phenomena, such as bank runs, technology adoption, currency crises, tax evasion or the emergence of cryptocurrencies.[1] In these models, actions are strategic complements in the sense

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[1]An incomplete list of models includes bank runs (Diamond and Dybvig, 1983; Postlewaite and Vives, 1987; Goldstein and Pauzner, 2005; Rochet and Vives, 2004), currency and balance-of-payments crises (Obstfeld, 1986, 1996; Morris and Shin, 1998), technology adoption (Dybvig and Spatt, 1983; Farrell and Saloner, 1985; Katz and Shapiro, 1985), tax evasion (Bassetto and Phelan, 2008), regime change (Angeletos et al., 2007; Edmond, 2013), crime waves (Bond and Hagerty, 2010), and blockchain economics (Abadi and Brunnermeier, 2018).

that, as the fraction of agents in the population taking a particular action grows, e.g., more depositors withdraw their money from a bank, so do the incentives of any single agent to take such an action.

A methodological challenge in these models is the fact that complementarities typically bring about multiplicity of equilibria, which might limit the ability to provide sharp predictions and develop specific recipes for economic policies and institutional reform. To address this challenge, the literature on *global games* (Carlsson and van Damme, 1993; Morris and Shin, 1998) has proposed an equilibrium selection mechanism based on the introduction of incomplete information in the form of idiosyncratic noise about some common payoff parameter or 'economic fundamental', e.g., a bank's liquid assets available to fend off a potential wave of deposit withdrawals. The global games selection has proved very appealing, especially in models with homogeneous agents. The reasons are twofold. First, this approach provides a tractable characterization of equilibrium that is in many cases robust to different specifications of the noise. Second, it leads to an intuitive interpretation of the selected equilibrium as being the risk dominant equilibrium in the underlying complete information game.

An important feature of many of the above coordination problems is that agents exhibit heterogeneous incentives that go beyond idiosyncratic differences in information about economic fundamentals. These differences are driven, for instance, by heterogeneity in financial portfolios, adoption costs or preferences (e.g., risk attitudes). Accordingly, the introduction of heterogeneity is useful to understand its interplay with strategic complementarities and how it can affect coordination. Moreover, accounting for agent heterogeneity is essential to do quantitative and empirical work with these models. However, while the global games selection can be extended to games with heterogeneous agents (Frankel, Morris and Pauzner, 2003), there exist only limited results on the *characterization* of the selected equilibrium specific to particular applications. Because of this, the economic content behind the selection and its robustness to the specification of noise are not fully understood.

This paper addresses these issues by analyzing the properties of the global games selection in canonical coordination games with binary actions played by heterogeneous agents. In the model each agent chooses and action $a_i \in \{0, 1\}$, and payoffs depend on the average action in the population—which reflects the fraction of agents taking action one, a common parameter θ and a private type that is heterogeneously distributed in the population. A higher type is associated with stronger incentives to take action one. Strategic complementarities stem from the fact that payoff differences between taking action one and action zero are increasing in the average action. In the global games version of the game agents do not observe the common parameter and instead receive a noisy signal of θ, which leads to equilibrium uniqueness. By taking the noise to zero the unique equilibrium converges to one of the Nash equilibrium of the game in which θ is common knowledge. This limit equilibrium represents the global games selection.

This paper makes several contributions. First, Section 2 introduces a key separability condition on payoffs that leads both to a tractable characterization of the global games selection and to its invariance with respect to the distribution of noise. Payoff separability means that the effect of the average action on payoffs must be symmetric (after some normalization) across player types, even though different types may exhibit different 'intrinsic' incentives to take a particular action. In addition, Section 3 identifies the necessary and sufficient conditions for the presence of multiple Nash equilibria in the complete information game. These are the conditions under which the global games selection, defined in Section 4, has bite.

Second, I define in Section 5 the notion of *ex ante risk dominance* for games with separable payoffs and show that the global games selection picks the ex ante risk dominant Nash equilibrium. Ex ante risk dominance has the following economic meaning. Consider an agent that, before learning her type, believes that she will always be the *marginal type*, i.e., the type that is indifferent between adopting ($a_i = 1$) and not adopting ($a_i = 0$) and that agents with higher types adopt while lower-type agents do not adopt. Such beliefs are pessimistic in the sense that the agent considers her to be the agent with the weakest incentives to take a particular action. The ex ante risk dominant strategy profile is then the Nash equilibrium that maximizes the ex ante expected payoffs of an agent with such pessimistic beliefs. I show that, in addition to providing economic content to the global games selection, ex ante risk dominance leads to a very tractable characterization of the selection rule based on the common payoff parameter. Specifically, it identifies the tipping points of θ at which the adoption rate, i.e., the average action, discontinuously jumps and pins down the magnitude of the jump. I also provide the conditions under which a unique tipping point exists, which depend on the trade-off between the degree of heterogeneity and the sensitivity of individual payoffs to the average action.

Finally, I show in Section 6 that payoff separability is a tight condition for the invariance of the global games selection. Specifically, I show that the selection is 'uniform' under payoff separability, i.e., is not affected by the distribution of noise, and that we can always find non-separable payoffs under which the selection changes with the distribution of noise. Combined, our results reveal that the lack of robustness of the global games selection is linked to the presence of asymmetric average action effects, and that such asymmetries also preclude us from having a tractable characterization and a clear economic meaning of the selection.

The characterization of the selection in terms of ex ante risk dominance and the role that payoff separability plays can be traced to a key property of agents' beliefs about the average action in the global game, first identified by Sakovics and Steiner (2012). This property implies that types associated with a tipping point, i.e., those who switch actions as their signals indicate that θ is at the tipping point, believe *on average* that the average action is uniformly distributed. Hence, under payoff separability we can average payoffs across types by taking the expectation of the effect of the average action using the uniform distribution, since this effect is the same for all players. I show that these average payoffs are equivalent to the ex ante expected payoffs of a player believing that she will always be the *marginal type*. This equivalence also illustrates why the uniform selection can fail without payoff separability: when average effects are not symmetric we no longer can use the uniform distribution to compute average payoffs of the types associated with a tipping point, since individual beliefs cannot be separated from payoffs and thus can no longer be averaged out. As I show in Section 6, these individual beliefs depend both on the payoff structure and on the distribution of noise, making the selection sensitive to the latter.

Related Literature. The paper is closely related to the work on global games with heterogeneous agents of Frankel et al. (2003), Sakovics and Steiner (2012) and Drozd and Serrano-Padial (2018). Frankel et al. (2003) propose the global games selection for games with heterogeneous payoffs and define the notion of uniform selection as being independent of the noise distribution. They show the existence and uniqueness of equilibrium and present characterization results for some classes of games that do not include the kind of coordination games studied here. Sakovics and Steiner (2012) study regime change games with heterogeneous payoffs and identify the mentioned condition of equilibrium beliefs, which they use to characterize the global games selection. Regime change models admit only symmetric equilibria in the global game, which imply the existence of a single tipping point that involves the whole population of players. Drozd and Serrano-Padial (2018) extend the characterization to games with asymmetric equilibria in the context of a model of credit enforcement frictions. I build on these papers by generalizing the characterization in Drozd and Serrano-Padial (2018) to canonical binary-action games with asymmetric equilibria, by providing economic content behind the selection in terms of ex ante risk dominance, and by identifying necessary and sufficient conditions for the selection to be uniform.

There are also a few recent papers that apply global games techniques to models with preference heterogeneity. For instance, Corsetti et al. (2004) analyze currency crises in a population with large and small traders. Abadi and Brunnermeier (2018) use the approach of Drozd and Serrano-Padial (2018) to study blockchain economics and the market for cryptocurrencies, while Dai and Yang (2018) look at the emergence of organizations when the distribution of types is uniform.[2]

2 Model

There is a continuum of players of measure one playing a two-action simultaneous game. Each player $i \in [0,1]$ chooses action $a_i \in \{0,1\}$. Player payoffs, denoted by $U(a_i, a, \theta, w)$, depend on own action a_i and on the average action a. The average action represents the mass of agents choosing action 1 and thus I also refer to it as the *adoption rate*. In addition, payoffs depend on a 'global' parameter $\theta \in \Theta \subseteq \mathbb{R}$, where Θ is a closed bounded interval, and on the player's type $w \in \mathbb{R}$, which is distributed in the population according to cumulative distribution function $F(w)$. F is continuous with support $[\underline{w}, \bar{w}]$. Both U and F are common knowledge.

Payoffs are continuous and bounded. Let $\Delta U(a, \theta, w) := U(1, a, \theta, w) - U(0, a, \theta, w)$ be the payoff difference between taking action 1 and action 0. I make the following assumption about payoffs.

[2]There exists a related literature on coordination in networks, e.g., Golub and Morris (2017) and Leister et al. (2018). In these models, due to the bilateral nature of the interaction between agents, beliefs are represented by expectations of the individual actions of each neighbor, rather than distributions over the average action.

Assumption 1. *Payoffs satisfy the following properties:*

(i) ΔU *is bounded, Lipschitz continuous and increasing in* a, θ *and* w. *That is,* U *exhibits increasing differences w.r.t.* a, θ *and* w.

(ii) There exist $\underline{\theta} > \inf \Theta$ *and* $\bar{\theta} < \sup \Theta$ *such that* $\Delta U(1, \theta, w) < 0$ *if* $\theta < \underline{\theta}$ *and* $\Delta U(0, \theta, w) > 0$ *if* $\theta > \bar{\theta}$ *for all* w.

Increasing differences with respect to the average action leads to strategic complementarities of players' actions, that is, a higher adoption rate strengthens incentives to adopt. Similarly, a higher θ increases the incentives to take action 1, while higher types w have stronger incentives to take action 1 than lower types. Condition (ii) involves the presence of dominance regions, that is, ranges of parameter values at which all player types have a strictly dominant strategy. Such a condition ensures that the global game version of the model has a unique equilibrium.

I next define payoff separability, which will play an instrumental role in both the characterization of equilibrium and the robustness of the global game selection to different specifications of noise.

Definition 1. *Payoffs are separable in* w *and* a *if there exist Lipschitz continuous and bounded functions* v_0, v_1, v_2 *and constant* $\xi > 0$ *such that*

$$U(a_i, a, \theta, w) = v_0(a_i, a, \theta)v_1(\theta, w) + v_2(a_i, \theta, w), \quad v_1(\theta, w) > \xi \text{ for all } \theta, w.$$

Payoff separability imposes a strong symmetry restriction on the effect of the average action on payoffs: a change in the average action leads to the same change in payoffs for all players, up to scaling up or down payoffs by a type-contingent scalar $v_1(\theta, w)$. Accordingly, separable payoffs are the sum of a common component and an idiosyncratic or private component, once payoffs are normalized.

Note that payoff separability leads to $a_i = 1$ yielding a higher payoff than $a_i = 0$ for given average action a if $u(a, \theta) + v(\theta, w) \geq 0$, where

$$u(a, \theta) := v_0(1, \theta, a) - v_0(0, a, \theta) \text{ and } v(\theta, w) := \frac{v_2(1, \theta, w) - v_2(0, \theta, w)}{v_1(\theta, w)}.$$

Under separable payoffs we can define normalized payoffs as

$$\tilde{U}(a_i, a, \theta, w) = v_0(a_i, a, \theta) + v_2(a_i, \theta, w)/v_1(\theta, w),$$

and normalized differences as

$$\Delta\tilde{U}(\theta, a, w) := \frac{\Delta U(\theta, a, w)}{v_1(\theta, w)} = u(a, \theta) + v(\theta, w).$$

I first analyze the game under complete information and then study its global game version. To ease exposition, all proofs are relegated to the Appendix.

3 Equilibrium of the Complete Information Game

The game is of complete information if θ is common knowledge. Let $\mathbf{a}$ denote the profile of strategies in the population. A Nash equilibrium (NE) of the complete information game is a strategy profile $\mathbf{a}^*$ satisfying

$$a_i^* = 1 \text{ if } \Delta U(a^*, \theta, w) \geq 0, \tag{1}$$

$$a_i^* = 0 \text{ if } \Delta U(a^*, \theta, w) \leq 0, \tag{2}$$

$$a^* = \int_0^1 a_i^* di. \tag{3}$$

A profile $\mathbf{a}$ is monotone if higher types exhibit higher actions. If a profile is monotone then it is characterized by its *marginal type*, i.e., the lowest type choosing $a_i = 1$. If w is the marginal type of profile $\mathbf{a}$ then the average action is given by $a = 1 - F(w)$.

Proposition 1. *Profile* $\mathbf{a}^*$ *is a NE of the complete information game iff it is monotone and satisfies one of the following conditions:*

1. $a^* = 1 - F(w^*)$, *where the marginal type* w^* *is a solution to*

$$\Delta U(1 - F(w^*), \theta, w^*) = 0; \tag{4}$$

2. $a^* = 0$ *with* $\Delta U(0, \theta, w) < 0$ *for all* w;

3. $a^* = 1$ *with* $\Delta U(1, \theta, w) > 0$ *for all* w.

The monotonicity of equilibrium follows from the fact that higher types have higher incentives to adopt, while equation (4) reflects that the marginal type is indifferent between adopting or not. Conditions 2) and 3) respectively represent symmetric equilibria in which no one and every one adopts. In what follows, I indistinctly use the word equilibrium to refer to profile $\mathbf{a}^*$ or to marginal type w^*.

Equilibrium multiplicity arises when (4) has more than one solution or when it has one solution and conditions 2) or 3) in Proposition 1 are satisfied. The next proposition identifies the necessary and sufficient conditions for the presence of multiple equilibria for a non-degenerate subset of Θ, that is, for a subset with positive Lebesgue measure.

Proposition 2. *There exists multiple NE for a non-degenerate set of θ if and only if there exists a pair (θ', w') such that $\Delta U(1-F(w'), \theta', w') = 0$ and $\Delta U(1-F(w), \theta, w)$ is strictly decreasing in w at (θ', w').*

The existence of multiple equilibria is modulated by two opposing forces shaping $\Delta U(1-F(w), \theta, w)$: heterogeneity and contagion. Heterogeneity implies that higher types have higher incentives to adopt at any level of the average action. Hence, a higher marginal type, keeping the average action constant, increases ΔU. Contagion is driven by the fact that a lower average action reduces the incentives to adopt of all types. Accordingly, as the marginal type w increases the average action decreases, pushing down ΔU. That is, as the marginal type goes up she has higher 'intrinsic' incentives to adopt but strategic complementarities dampen such incentives due to a lower adoption rate in the population. In this context, the heterogeneity of types embodied in F affects the strength of strategic complementarities by how fast the average action decreases with the marginal type. The faster it goes down the more likely $\Delta U(1 - F(w), \theta, w)$ is to be decreasing. The intuition is straightforward: multiple equilibria are due to contagion among heterogeneous agents, and these effects are stronger when they involve a sufficiently high mass of players that are not too heterogeneous.

The following example illustrates the effect of heterogeneity.

Example 1 (Investment game)**.** *Players decide whether to invest ($a_i = 1$) or not ($a_i = 0$). The payoff from investing is $u(1, a, \theta, w) = \theta(1 + a + w^2)$ with $w \in [0, 1]$, while the payoff from not investing is always 1, i.e., $\Delta U(a, \theta, w) = \theta(1+a+w^2) - 1$.*

Notice that for $\theta > \bar{\theta} = 1$ investing is a dominant strategy for all types, while not investing is a dominant strategy if $\theta < \underline{\theta} = \frac{1}{3}$. Consider two alternative type distributions: uniform ($F(w) = w$) and quadratic ($F(w) = w^2$).

Under the uniform distribution, $\Delta U(1-F(w), \theta, w) = 2\theta - 1 - \theta(w - w^2)$, which is strictly convex in w for $\theta > 0$ and reaches a minimum at $w = 0.5$ and a maximum at $w \in \{0, 1\}$. Hence, by Proposition 2, there are multiple equilibria in an interval of θ, specifically for all $\theta \in \left[\frac{1}{2}, \frac{4}{7}\right]$. Figure 1 depicts $\Delta U(1-F(w), \theta, w)$ for $\theta = \frac{9}{16}$.

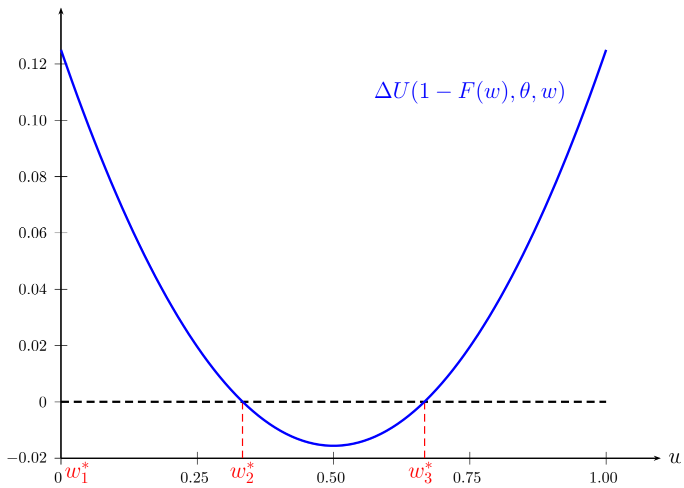

Figure 1: Multiplicity of Equilibria

The figure shows the existence of three equilibria. The first equilibrium corresponds to full adoption, i.e., $w_1^* = 0$ since $\Delta U(1, \theta, 0) > 0$. The other two correspond to the solutions of $\Delta U(1 - F(w^*), \frac{9}{16}, w^*) = 0$, which are $w_2^* = \frac{1}{3}$ and $w_3^* = \frac{2}{3}$, and exhibit adoption rates $1 - F(w_2^*) = \frac{2}{3}$ and $1 - F(w_1^*) = \frac{1}{3}$, respectively. The convexity of $\Delta U(1 - F(w), \theta, w)$ is driven by the fact that under the uniform distribution, the average action decreases initially faster and eventually slower than the *intrinsic* payoff from adoption of the marginal type, given by w^2. It is worth noting that, of the three equilibria in Figure 1, only the high adoption (w_1^*) and the low adoption (w_3^*) equilibria are stable.[3]

In contrast, under the quadratic distribution, $\Delta U(1 - F(w), \theta, w) = 2\theta - 1$ for all w. Hence, by Propositions 1 and 2 there is a unique equilibrium for any $\theta \neq 0.5$. If $\theta > 0$ the unique equilibrium involves $a^* = 1$ since $\Delta U(1 - F(w), \theta, w) > 0$, while $a^* = 0$ if $\theta < 0.5$. The reason behind uniqueness is that F concentrates its mass

[3] If the adoption rate moves away from a_2^* by an infinitesimal amount, standard dynamics would lead to one of the two other equilibria. This is because $\Delta U(1 - F(w), \theta, w) > 0$ on (w_1^*, w_2^*) so agents with types in this interval would switch to $a_i = 1$ if the adoption rate raises above $1 - F(w_2^*)$. Likewise, types $w > w_2^*$ would switch to $a_1 = 0$ if the adoption rate falls below $1 - F(w_2^*)$.

precisely where incentives grow at a faster rate, i.e., at high values of w. Hence, at types where incentives grow slow (fast) the average action is also decreasing slowly (fast), perfectly offsetting each other.

4 The Global Games Selection

The Global Games (GG) approach to resolve equilibrium indeterminacy (Frankel et al., 2003) is based on introducing incomplete information about parameter θ. Specifically, each agent gets a noisy signal $s = \theta + \nu\eta$, where $\nu > 0$ is the noise scale parameter, and η is independently distributed according to continuous distribution H_w with full support on $[-1/2, 1/2]$ and density h_w, which is allowed to depend on the agent's type. We assume the the exact LLN applies within type. A player strategy in the global game is a mapping $a_i(s)$ from signals to actions in $\{0, 1\}$.

The idiosyncratic nature of signal noise prevents agents to use their private signals as correlation devices to coordinate their actions, leading to a unique equilibrium. Moreover, since agents use their signals to extract information about θ, the average action in the selected equilibrium is increasing in θ, leading to a selection rule driven by "economic fundamentals", i.e., common payoff parameters.

The goal of the GG selection is to induce uniqueness of Bayes Nash equilibrium in the game with incomplete information and then select an equilibrium of the complete information game by taking the limit as $\nu \to 0$. For convenience, I assume that agents believe θ is uniformly distributed in Θ, which leads to agents' posteriors that also are uniform. I discuss below how relaxing this assumption does not substantively alter our results. This is because any well-behaved prior leads to a posterior distribution that approximates the uniform distribution as its support collapses into the actual value of θ when $\nu \to 0$ (Frankel et al., 2003).

I first establish that there is a unique equilibrium in the global game for sufficiently small noise.

Proposition 3. *There exists $\bar{\nu} > 0$ such that for all $\nu < \bar{\nu}$ there is essentially a unique equilibrium. Moreover, equilibrium strategies follow a cutoff rule:*

$$a_i(s) = \begin{cases} 0 & s < k^{\nu}(w) \\ 1 & s \geq k^{\nu}(w), \end{cases} \tag{5}$$

where k^ν satisfies the system of indifference conditions

$$E[\Delta U(a, \theta, w)|k^\nu; s = k^\nu(w)] = 0 \text{ for all } w \in [\underline{w}, \bar{w}], \tag{6}$$

and $E[\Delta U(a, \theta, w)|k^\nu, s]$ denotes expected payoff differences conditional on receiving signal s when all players use cutoff strategies given by k^ν.

The proof is based on standard arguments in the global games literature and is adapted from Drozd and Serrano-Padial (2018). First, the fact that payoffs exhibit increasing differences w.r.t. signals and actions implies that the global game is a supermodular game. From existing results on supermodular games (Milgrom and Roberts, 1990; Vives, 1990; Van Zandt and Vives, 2007), we know that the game has both a least and a largest equilibrium. Moreover, players follow monotone strategies in these equilibria, that is, each player uses a signal cutoff $k^\nu(w)$ above which they choose action $a_i = 1$. Second, I show that shifting up all cutoffs by the same amount does not affect agents' beliefs about adoption rates when they receive their cutoff signal, while it does lead to higher expectations about θ. I exploit this fact to prove that, as we move up cutoffs from the least to the largest equilibrium, expected payoffs differences go up and thus there can be only one equilibrium satisfying indifference conditions (6) (up to differences in behavior at cutoff signals).

As Frankel et al. (2003) point out, the translation invariance of agents' beliefs about adoption rates is driven by the uniform prior and the independent, additive noise. Together they imply that an agent with signal s believes that θ is uniformly distributed in $[s - \nu/2, s + \nu/2]$ and that other agents' signals fall in the interval $[s-\nu, s+\nu]$. Hence, as cutoffs are shifted, an agent receiving her cutoff signal believes the support of θ and thus the distribution of other agents' signals shift by the same amount as everyone's cutoff. But since the adoption rate is given by the mass of agents with signals above their respective cutoffs, the agent's beliefs about adoption rates, conditional on receiving her cutoff signal, do not change.[4]

A desirable property of the GG selection is to be robust to different noise distributions H_w, given that the goal is to pin down equilibrium in the limit and, hence, the introduction of noise is just a convenient technical device to introduce miscoordination risk. In such a case, the GG selection is said to be *uniform*.

[4]With a non-uniform prior, translation invariance holds approximately for ν sufficiently small.

5 Characterizing the Global Games Selection

In this section I introduce the notion of ex ante risk dominant equilibrium in the complete information game and show that it coincides with the GG selection under payoff separability. By doing so I am able to (i) characterize the selected equilibrium without relying on the introduction of incomplete information, and (ii) provide an economic meaning behind the GG selection under heterogeneity, which has thus far remained an open question in the literature. In the next section I show that payoff separability is a tight condition for the GG selection to be uniform. Combined, these results imply that the symmetry of average-action effects implied by separable payoffs links the robustness of the GG selection to a natural economic interpretation of the selection rule.

5.1 Preliminaries: Games with Homogeneous Payoffs

In order to introduce the notion of ex ante risk dominance and its connection to the GG selection, it is useful to first analyze the case in which there is no payoff relevant heterogeneity, that is, $U(a_i, a, \theta, w) = U(a_i, a, \theta)$ and $\Delta U(a, \theta, w) = \Delta U(a, \theta)$ for all w. In this context, one can extend the traditional notion of risk dominance in 2×2 games to the games with a continuum of players studied here. Specifically, in a 2×2 game a NE is *risk dominant* if the equilibrium strategy of each player maximizes her payoff given her belief that her opponent chooses her strategy randomly. We can apply the same type of *Laplacian beliefs* to the average action in the population to arrive to the following definition of risk dominance.

Definition 2. *A NE* $\mathbf{a}^*$ *in the homogeneous payoff game is risk dominant if* a^* *maximizes player i's payoffs when she believes that the average action is uniformly distributed, i.e.,*

$$a_i^* = 1 \text{ if } \int_0^1 \Delta U(a, \theta) da \geq 0.$$

Under homogeneous payoffs it is easy to check that the game has two symmetric equilibria for all $\theta \in [\underline{\theta}, \bar{\theta}]$, respectively involving zero adoption ($a_i^* = 0 \ \forall i$) and full adoption ($a_i^* = 1 \ \forall i$). There is also an unstable partial adoption equilibrium.[5]

[5] In this case a^* satisfies $\Delta U(a^*, \theta) = 0$ and all agents are indifferent between adopting or not.

The fact that $\Delta U(a,\theta)$ is increasing in θ implies that the no adoption equilibrium will be risk dominant at low values of θ, while the full adoption equilibrium will be risk dominant at high values. Accordingly risk dominance leads to the following selection rule:[6]

$$a_i^{RD} = \begin{cases} 0 & \theta < \hat{\theta} \\ 1 & \theta \geq \hat{\theta} \end{cases}, \quad \text{where } \hat{\theta} \text{ solves } \int_0^1 \Delta U(a,\hat{\theta})da = 0. \tag{7}$$

Now consider the GG selection and, for simplicity, assume that the distribution of noise is the same for all types. It turns out that, as Morris and Shin (2003) illustrate, the RD equilibrium coincides with the GG selection, that is, the signal cutoff in (5) satisfies $k(w) = \hat{\theta}$ in the limit as $\nu \to 0$.

Such an equivalence is brought by the additivity and independence of noise: because of payoff homogeneity, all players use the same signal cutoff in equilibrium. Hence, an agent receiving the signal cutoff $s = k^\nu(w)$, who is indifferent between the two actions, believes that all agents with lower signals than s do not adopt while those with higher signals adopt. That is, this agent believes that the aggregate action is $a = 1 - Prob(s' < s)$, which is equal to the mass of agents with noise terms above her own noise term η: $a = 1 - H(\eta)$. Since the agent does not observe η, she deems $H(\eta)$ and thus a as random variables that are uniformly distributed. This is known as the Laplacian property in global games (Morris and Shin, 2003), and leads to the above cutoffs given that $s \to \theta$ as $\nu \to 0$ and thus

$$\lim_{\nu \to 0} E[\Delta U(a,\theta) | k^\nu, k^\nu(w)] = \int_0^1 \Delta U(a,\theta)da.$$

An ex ante interpretation. Selection rule (7) can be rewritten as choosing the strategy profile that maximizes expected payoffs under Laplacian beliefs, given by

$$\int_0^1 \mathbf{1}_{\{a_i=0\}} U(0,a,\theta)da + \int_0^1 \mathbf{1}_{\{a_i=1\}} U(1,a,\theta)da. \tag{8}$$

Using the change of variable $a = 1 - F(w)$ and focusing only on monotone NE

[6] Since $\theta = \hat{\theta}$ is a zero probability event, I assume without loss of generality that the full adoption equilibrium is selected in the degenerate case in which both equilibria are risk dominant.

we can express the above expected payoffs as follows:[7]

$$\int_{\underline{w}}^{w^*} U(0, 1-F(w), \theta)dF(w) + \int_{w^*}^{\bar{w}} U(1, 1-F(w), \theta)dF(w). \tag{9}$$

This expression leads to the following *ex ante* interpretation of the risk dominant equilibrium: it is the monotone strategy profile that maximizes the ex ante expected payoff of an agent, i.e., before learning her type, that believes her to always be the *marginal type*. That is, she believes she is the highest type whenever she chooses $a_i = 0$ (first integral in (9)), and the lowest type when she chooses $a_i = 1$.

Equipped with this interpretation I now introduce the notion of ex ante risk dominance when heterogeneity is payoff relevant.

5.2 Ex Ante Risk Dominance under Payoff Separability

I define ex ante risk dominance for games with separable payoffs. As illustrated in the case of homogeneous payoffs, it is based on the ex ante belief that a player will always be the marginal type. These beliefs are *pessimistic* in the sense that, while the player is aware of the heterogeneity of incentives and thus expects people with stronger incentives to adopt whenever she finds adoption optimal, she focuses on the "worst case scenario" in which she is *always* the agent with the weakest incentives to choose a particular action (i.e., the highest type not adopting or the lowest type adopting). A monotonote strategy profile is *ex ante* risk dominant (RD) if it maximizes the normalized expected payoffs of a player endowed with these beliefs. The restriction to monotone profiles embodies the belief that higher types have a stronger incentive to adopt.

Definition 3 (Ex ante risk dominance)**.** *A strategy profile is ex ante risk dominant if it is monotone and its marginal type solves*

$$\max_{\hat{w} \in [\underline{w}, \bar{w}]} \int_{\underline{w}}^{\hat{w}} \tilde{U}(0, 1-F(w), \theta, w)dF(w) + \int_{\hat{w}}^{\bar{w}} \tilde{U}(1, 1-F(w), \theta, w)dF(w). \tag{10}$$

Notice that, while this notion can be extended to non-separable payoffs, it is robust to affine transformations of payoffs only when U is separable in w and a. The

[7] Since types are payoff irrelevant, there exist NE with partial adoption that are not monotone. Since they are never risk dominant this assumption is without loss.

reason is that any comparison of strategy profiles that relies on ex ante expected payoffs depends on the cardinal comparison of payoffs across types. In other words, multiplying player payoffs by some non-constant function of types $v'(w) > 0$ changes the weighted average of payoffs given by (10). While the above definition takes care of this issue by focusing on normalized payoffs, without payoff separability there does not exist a natural way to normalize payoffs.

The next lemma shows that ex ante RD profiles are NE and presents alternative characterizations of ex ante RD. From now on, I refer to a NE by its marginal type w^*, and denote $W^*(\theta)$ the set of NE marginal types associated with parameter θ.

Lemma 1. *If a monotone profile with marginal type $\hat{w}$ is ex ante RD then $\hat{w} \in W^*(\theta)$. In addition, the following statements are equivalent:*

i) w^* is ex ante RD;

ii) w^* maximizes

$$\int_0^{1-F(w^*)} \tilde{U}(1, a, \theta, F^{-1}(1-a))da + \int_{1-F(w^*)}^{1} \tilde{U}(0, a, \theta, F^{-1}(1-a))da. \tag{11}$$

iii) for all $w^{*'} \in W^*(\theta)$,

$$\int_{w^*}^{w^{*'}} \Delta\tilde{U}(1-F(w), \theta, w)dF(w) \geq 0; \tag{12}$$

iv) for all $w^{*'} \in W^*(\theta)$,

$$\int_{1-F(w^{*'})}^{1-F(w^*)} u(a, \theta)da + \int_{w^*}^{w^{*'}} v(\theta, w)dF(w) \geq 0. \tag{13}$$

Being a NE directly follows from (10): if a monotone profile is not a NE then Proposition 1 is violated and thus we can always increase or decrease its marginal type so that the objective function in (10) goes up. Condition (ii), which follows from the change of variable $a = 1 - F(w)$, provides an alternative definition of ex ante RD: it is the monotone profile that maximizes the *sum* of payoffs of the marginal types when the average action is assumed to be uniformly distributed. This leads to the alternative interpretation of ex ante RD: a player believes, before learning her type, that she will always be the marginal type but faces complete uncertainty about adoption rates and hence applies the principle of insufficient reason and deems all adoption rates equally likely. Condition (iii) is just a convenient rewriting of Definition 3, since it helps identify the ex ante RD equilibrium by looking at the expected value of $\Delta\tilde{U}(1-F(w),\theta,w)$ between the marginal types of different NE. Finally, condition (iv) directly follows from condition (iii) after substituting for $\Delta\tilde{U}(1-F(w),\theta,w) = u(1-F(w),\theta)+v(\theta,w)$ and applying the change in variable $a = 1-F(w)$. To interpret this condition consider two NE with marginal types w^* and $w^{*'} > w^*$. The condition states that, when comparing the two NE, we should look at the sum of two terms: the expected value of the symmetric average-action effect on payoff differences assuming that the average action is uniformly distributed in $[1-F(w^{*'}), 1-F(w^*)]$; and the average private component of payoff differences for types between w^* and $w^{*'}$. It turns out that, as I show below, (iv) coincides with the conditions pinning down the global games selection.

Proposition 4. *An ex ante RD equilibrium exists and is generically unique, that is, the set of θ for which there is more than one ex ante RD equilibrium has zero Lebesgue measure.*

To illustrate how to pin down the ex ante RD profile using condition (iii) in Lemma 1, consider Example 1 when F is the uniform distribution. Recall that there are three equilibria with marginal types for $\theta \in \left[\frac{1}{2}, \frac{4}{7}\right)$. The green shaded area in Figure 2 represents the range of types at which $\Delta\tilde{U}(1-F(w),\theta,w)$ is positive between two NE, while the red area is associated with $\Delta\tilde{U}(1-F(w),\theta,w) < 0$.

First, notice that the (unstable) NE with marginal type w_2^* is not ex ante RD. Moving to the (stable) equilibrium w_1^* or to w_3^* increases the objective function in (10) by either adding area A or by subtracting area B. Second, to determine whether w_1^* or w_3^* are ex ante RD we need to compare the size of area A given the distribution of types $F(w)$ to area B. If A is the larger area then w_1^* is ex ante RD and otherwise w_3^* is the ex ante RD equilibrium. Since $\Delta U(1-F(w),\theta,w)$ is increasing in θ, area A gets bigger at higher θ while area B shrinks. Hence, we can find a cutoff value $\hat{\theta}$ above which w_1^* is the ex ante RD and below which w_3^*. This cutoff value $\hat{\theta}$ equates the sizes of areas B and C, i.e., it is the solution to

$$\int_0^{w_3^*(\theta)} \Delta U(1-F(w),\theta,w)f(w)dw = \int_0^{w_3^*(\theta)} \left(2\theta - 1 - \theta(w-w^2)\right)dw = 0, \quad (14)$$

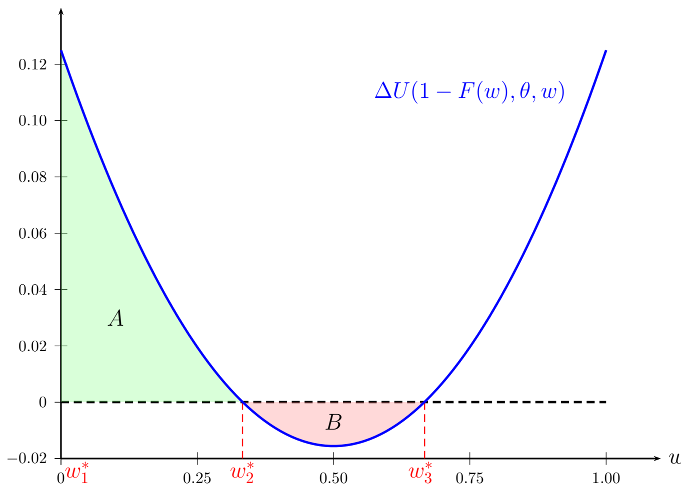

Figure 2: Finding the ex ante RD equilibrium

where $w_3^*(\theta)$ is the largest solution of $\Delta U(1 - F(w), \theta, w) = 2\theta - 1 - \theta(w - w^2) = 0$. This leads to $\hat{\theta} \approx 0.552$ and $w_3^*(\hat{\theta}) = \frac{3}{4}$. Figure 3 plots the marginal type (w^*_{RD}) and the adoption rate (a^*_{RD}) of the ex ante RD equilibrium for different values of θ.

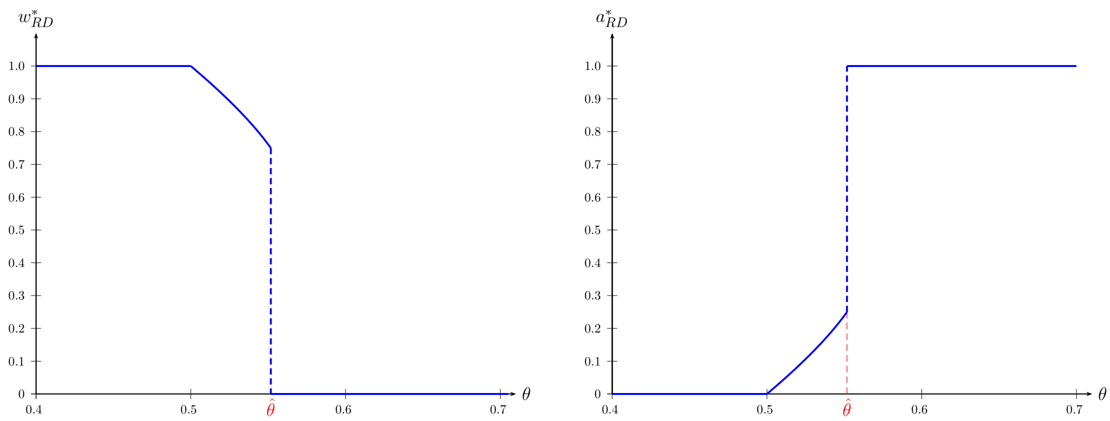

Figure 3: Marginal type (left) and adoption rate (right) in the ex ante RD eq.

This example illustrates the behavior of the selection rule based on ex ante RD as a function of the global parameter θ, which exhibits a tipping point at $\hat{\theta}$ associated with a jump in adoption rates. The next proposition generalizes the example by characterizing the ex ante RD as a function of the distribution of types and the payoff structure. The characterization provides a tractable way to pin down the tipping points associated with jumps in adoption rates. Specifically, the proposition formally establishes that the marginal type w^*_{RD} is (weakly) decreasing and varies continuously with θ at all θ at which the ex ante RD equilibrium is unique. Importantly, at every θ at which there are multiple ex ante RD equilibria, an infinitesimal increase in θ causes a discontinuous drop from the highest to the lowest w^*_{RD} associated with such θ. In the example depicted in Figure 3, w^*_{RD} drops from $\frac{3}{4}$ to 0 as θ crosses the threshold $\hat{\theta}$.

First, I provide a formal definition of the ex ante RD selection, which assumes that the lowest marginal type is picked at the set of θ exhibiting multiple ex ante RD equilibria. This assumption simplifies the statement of the result and is innocuous since such a set has zero measure by Proposition 4.

Definition 4. *The ex ante RD selection is the mapping* $w_{RD} : \Theta \to [\underline{w}, \bar{w}]$ *given by*

$$w_{RD}(\theta) = \min\{w^* \in W^*(\theta) : w^* \textit{ is ex ante RD}\}. \tag{15}$$

In addition, let $w^+_{RD}(\theta) = \limsup_{\theta' \to \theta} w_{RD}(\theta')$.

Proposition 5. *The function* w_{RD} *is well-defined, decreasing and right-continuous. Moreover, if there are multiple NE for a non-degenerate subset of* θ, *then there exists a unique collection of parameter thresholds* $\theta_1 < \theta_2 < \cdots < \theta_n$ *such that:*

1. w_{RD} *is continuous at all* $\theta \neq \theta_j$, $j = 1, \cdots, n$;

2. w_{RD} *is discontinuous at* θ_j, $j = 1, \cdots, n$, *where* θ_j *satisfies*

$$\max_{w^* \in W^*(\theta_j), w^* < w^+_{RD}(\theta_j)} \int_{w^*}^{w^+_{RD}(\theta_j)} \Delta\tilde{U}(1 - F(w), \theta_j, w)dF(w) = 0, \tag{16}$$

   and w_{RD} *drops from* $w^+_{RD}(\theta_j)$ *to the smallest maximizer of the LHS of* (16);

3. $w_{RD}(\theta) = \underline{w}$ *if*

$$\max_{w^* \in W^*(\theta)} \int_{\underline{w}}^{w^*} \Delta\tilde{U}(1 - F(w), \theta, w)dF(w) \geq 0; \tag{17}$$

4. $w_{RD}(\theta) = \bar{w}$ *if* $\Delta\tilde{U}(1 - F(w), \theta, w) < 0$ *for all* $w < \bar{w}$ *or*

$$\max_{w^* \in W^*(\theta), w^* < \bar{w}} \int_{w^*}^{\bar{w}} \Delta\tilde{U}(1 - F(w), \theta, w)dF(w) < 0. \tag{18}$$

Applying condition (iv) in Lemma 1 leads to the following analytic characterization of the tipping points of the RD selection.

Corollary 1. *The collection* $\{\theta_j\}_{j=1}^{n}$ *defined in Proposition 5 satisfies*

$$\int_{1-F(w_{RD}^{+}(\theta_j))}^{1-F(w_{RD}(\theta_j))} u(a,\theta_j)da + \int_{w_{RD}(\theta_j)}^{w_{RD}^{+}(\theta_j)} v(\theta_j,w)dF(w) = 0. \tag{19}$$

Before showing the equivalence between the ex ante RD and the GG selection,I provide conditions under which there is a unique tipping point.

Corollary 2. *If* $\Delta\tilde{U}(1-F(w),\theta,w)$ *is strictly quasiconcave or decreasing in* w *for all* $\theta \in [\underline{\theta},\bar{\theta}]$ *then* w_{RD} *has a single discontinuity* θ_j*, which satisfies* (19).

The quasiconcavity or decreasingness of $\Delta\tilde{U}(1-F(w'),\theta',w')$ implies that there are at most three NE, as is the case in Example 1 and illustrated in Figure 1. Following the intuition conveyed with Figure 2, this leads to a single switch from the lowest to the highest adoption equilibrium, that is, to a single tipping point. This result can be useful for doing empirical or quantitative work with models where heterogeneity is given by unimodal type distributions, which are commonly used in practice. The reason is that single-peaked distributions, by concentrating mass around its peak, make $1-F(w)$ go down fast at w close to the peak and slow at w far away from the peak, potentially causing payoff differences to be quasiconcave.

5.3 The Equivalence Result

I next provide our first main result, namely, that the GG selection selects the ex ante RD equilibrium. The following proposition provides a characterization of the limit equilibrium cutoffs $k(w) = \lim_{\nu\to 0} k^{\nu}(w)$ in the global game. It generalizes the characterization of the limit equilibrium put forward by Drozd and Serrano-Padial (2018), who do so in the context of a model of credit enforcement.

Proposition 6. *Under payoff separability, there exists a unique collection of types* $\underline{w} = w_1 < w_2 < \cdots < w_I = \bar{w}$*, such that* $k(w)$ *alternates between being strictly decreasing and constant in intervals* (w_i, w_{i+1})*,* $i = 1, \cdot, I-1$*, and satisfies*

1. *if* $k(\cdot)$ *is strictly decreasing in* (w_i, w_{i+1}) *then* $\Delta\tilde{U}(1-F(w),k(w),w) = 0$ *for all* $w \in (w_i, w_{i+1})$*;*

2. if $k(w) = k_i$ in $[w_i, w_{i+1}]$ for some constant k_i then

$$\int_{1-F(w_{i+1})}^{1-F(w_i)} u(a, k_i)da + \int_{w_i}^{w_{i+1}} v(k_i, w)dF(w) = 0. \tag{20}$$

Moreover, the collection of types $\{w_i\}_{i=1}^{I}$ satisfies

$$\Delta\tilde{U}(1 - F(w_i), k(w_i), w_i) = 0, \quad i = 2, \cdots, I-1; \tag{21}$$

$$\Delta\tilde{U}(1 - F(w_1), k(w_1), w_1) \geq 0; \tag{22}$$

$$\Delta\tilde{U}(1 - F(w_I), k(w_I), w_I) \leq 0. \tag{23}$$

The characterization of limit cutoffs allows us to establish the equivalence between the global games selection and the notion of ex ante risk dominance.

Definition 5. *The global games selection is the mapping $w_{GG} : \Theta \to [\underline{w}, \bar{w}]$ defined by $w_{GG}(\theta) := \min\{w : k(w) = \theta\}$.*[8]

Corollary 3. *If payoffs are separable then $w_{GG}(\theta) = w_{RD}(\theta)$ for all $\theta \in \Theta$. That is, a NE is the limit equilibrium in the global game if and only if it is ex ante risk dominant.*

The proof logic is as follows. When $k(w)$ is strictly decreasing, an agent receiving $s = k(w)$ knows that every other agent is also getting signals in a neighborhood of s, and hence almost all agents with types below w choose $a = 0$ and almost all with types above w choose $a = 1$. Hence, in the limit the signal reveals θ and the agent perfectly anticipates the average action to be $1-F(w)$. Since she is indifferent between the two actions when she receives her signal cutoff, the latter must satisfy

$$\lim_{\nu \to 0} \mathbb{E}(\Delta\tilde{U}(\theta, a, w)|k^{\nu}, s = k^{\nu}(w)) = \Delta\tilde{U}(k(w), 1 - F(w), w) = 0.$$

In contrast, when $k(w) = k_i$ is constant in an interval (w_i, w_{i+1}), the agent faces uncertainty about the fraction of agents with types in the interval that got signals above their cutoffs, even in the limit. In particular, the agent is only certain that the average action must be within $1 - F(w_{i+1})$ and $1 - F(w_i)$. Accordingly, to solve

[8] I make the assumption, as I do in the definition of the ex ante RD selection, that the lowest marginal type is picked at the set of θ where the global game exhibits multiple limit equilibria.

the indifference conditions of types in (w_i, w_{i+1}) we need to pin down her individual beliefs, which depend on the payoff structure, the noise distribution and the distribution of types. Nonetheless, since both their signals and their cutoffs converge to the same limit as $\nu \to 0$, we can utilize the *average* indifference condition of types in (w_i, w_{i+1}) to pin down k_i. Due to continuity, the limit indifference condition of type $w \in (w_i, w_{i+1})$ can be shown to be

$$\int_0^1 \Delta\tilde{U}(1 - F(w_{i+1}) + z(F(w_{i+1}) - F(w_i)), k_i, w) dA_w(z|k; (w_i, w_{i+1})) = 0, \quad (24)$$

where $A_w(z|k; W')$ is the cdf representing the individual belief of an agent of type w conditional on $s = k(w)$ about the probability that the average action of types in W' is less than z.

Averaging the above indifference condition over types in (w_i, w_{i+1}) turns out to be key to recover k_i under payoff separability because of a key property of beliefs uncovered by Sakovics and Steiner (2012). They show that the *average conditional belief* about the average action is the uniform distribution. They call this property the *belief constraint*. Moreover, as the next lemma establishes, the belief constraint also applies if we restrict attention to any measurable subset of types W'.

Lemma 2 (Belief Constraint)**.** *Given any measurable subset of types W' and any cutoff function $\kappa : [\underline{w}, \bar{w}] \to [\inf \Theta + \nu/2, \sup \Theta - \nu/2]$,*

$$\int_{W'} A_w(z|\kappa; W') dF(w|w \in W') = z \text{ for all } z \in [0, 1]. \quad (25)$$

But notice that payoff separability requires that the effect of the average action on payoffs is symmetric across types, since $\Delta\tilde{U}(\theta, a, w) = u(a, \theta) + v(\theta, w)$. Hence, the averaging of normalized payoff differences over types in (24) can be separated into the averaging of beliefs about a and the averaging of the private component of payoff differences $v(\theta, w)$, yielding

$$\int_{w_i}^{w_{i+1}} \left[\int_0^1 u(a, k_i) dA_w(z|k; (w_i, w_{i+1})) \right] dF(w|w \in (w_i, w_{i+1}))$$
$$+ \int_{w_i}^{w_{i+1}} v(k_i, w) dF(w|w \in (w_i, w_{i+1})).$$

Since $u(\cdot)$ does not depend on w, we can interchange the limits of integration in the first term and replace the average belief with the uniform distribution, leading to the equilibrium condition (20) in Proposition 6.

It is worth pointing out that the uniform prior facilitates both the exposition and the proofs but it is not instrumental to obtain the results in the paper. As Frankel et al. (2003) have shown, any well-behaved prior with full support on Θ leads to a posterior distribution that is approximately uniform as $\nu \to 0$. Accordingly, I could drop the uniform prior assumption and modify Proposition 6 to state that limit equilibrium cutoffs are close to the cutoffs characterized in the proposition.

5.3.1 Games of Regime Change

Before turning to the question of when the GG selection is sensitive to the specification of noise, it is worth emphasizing that the equivalence between the GG selection and ex ante RD readily extends to games of regime change. In those games, there is a common threshold on the average adoption $\hat{a}(\theta)$ above which everyone finds $a_i = 1$ optimal and below which choosing $a_i = 0$ is a best response for any player, regardless of her type w. That is, payoff differences take the following form:

$$\Delta U(a, \theta, w) = \begin{cases} \overline{u}(a, \theta, w) & a > \hat{a}(\theta) \\ \underline{u}(a, \theta, w) & a < \hat{a}(\theta), \end{cases}$$

where $\overline{u}$ is a positive function, $\overline{u}$ is a negative function, and both are increasing in all their arguments, Lipschitz continuous and bounded.

This class of games always have a symmetric NE in which either everyone or no one adopts action one. Both equilibria coexist for a range of θ. This is the class of games studied by Sakovics and Steiner (2012). They show that, under payoff separability, the GG selection is given by a single limit cutoff k_1, followed by all types in $[\underline{w}, \bar{w}]$, satisfying condition (20).[9] Consequently, their result directly implies that the equivalence between the GG selection and ex ante RD also applies to games of regime change.[10]

[9] I refer the reader to the online Appendix of their paper: https://assets.aeaweb.org/assets/production/articles-attachments/aer/data/dec2012/20091318_app.pdf.

[10] It is straightforward to extend Propositions 4 and 5 to include the above class of payoffs, given that the fact that the selected equilibrium is symmetric greatly simplifies the characterization of the ex ante RD selection.

6 Uniform Selection

Payoff separability is not only behind the characterization of the GG selection and its ex ante risk dominance interpretation, but also guarantees that the GG selection is uniform. The latter is reflected in Proposition 6, given that the conditions pinning down the equilibrium cutoffs are independent of the noise distribution.

The intuition behind uniform selection when payoffs are separable is as follows. Notice that the limit cutoff is pinned down by the average indifference condition of those types that follow the same limit cutoff and that the belief constraint implies that, while the density associated with the individual beliefs about the average action a may vary with H_w, the average density does not. Hence, finding the common cutoff is akin to compute a weighted average of the effect of the average action a on payoffs when the sum of weights is constant across noise distributions. When payoffs are separable, the average action a has the same effect on payoffs across agent types, so taking the average effect of a across types involves taking a weighted average of a constant. Hence, since weights vary with H_w but its sum does not, the average effect of a is invariant to the distribution of noise.

In contrast, when payoffs are not separable, the effect of a is no longer constant across types, and thus the dependence of individual weights on the noise distribution can make the average effect also dependent on the noise distribution. In turn, this leads to the limit cutoff to vary across noise distributions.

I formally show how payoff separability is instrumental to the robustness of the GG selection through a series of results. First, I provide a characterization of the individual beliefs of any given type about the average action of any other type. Equipped with these individual beliefs, I then show that individual beliefs about the average action *in the population* differ across noise distributions. Finally, I prove by constructing an example that the lack of payoff separability can lead to the GG selection not being uniform, i.e., to vary with the distribution of noise.

The next lemma characterizes individual beliefs of type w about the average action $a_{w'}$ of type w', as a function of the noise distribution and the normalized difference in signal cutoffs $\Delta k(w, w') = \frac{k^\nu(w') - k^\nu(w)}{\nu}$.

Lemma 3. *Let $z \sim U[0,1]$. An agent of type w receiving $s = k^\nu(w)$ believes that*

1. *if $w' = w$ then $a_{w'} = z$;*

2. *if $w' < w$ then*

$$a_{w'} = \begin{cases} 0 & z < 1 - H_w(\frac{1}{2} - \Delta k(w,w')) \\ 1 - H_{w'}(H_w^{-1}(1-z) + \Delta k(w,w')) & \textit{otherwise}; \end{cases}$$

3. *if $w' > w$ then*

$$a_{w'} = \begin{cases} 1 - H_{w'}(H_w^{-1}(1-z) + \Delta k(w,w')) & z < 1 - H_w(-\frac{1}{2} - \Delta k(w,w')) \\ 1 & \textit{otherwise}. \end{cases}$$

In words, the lemma establishes that an agent with type w believes that her own-type average action is uniformly distributed, and that the average action of a type w' with $\Delta k(w,w') \neq 0$ is fully determined by her own type average action. The former is the Laplacian property, while the latter is driven by the exact LLN.

To gain some intuition consider the following facts. First, when a type-w agent receives her cutoff signal $k^\nu(w)$, the noise term in the signal, given by $\eta = (k^\nu(w) - \theta)/\nu$, represents the cutoff for adoption of agents of her own type. That is, same-type agents with $\eta' > \eta$ adopt and, by the exact LLN, their adoption rate is given by $z = 1 - H_w(\eta)$. But since noise terms are i.i.d. within type, z is uniformly distributed. Now consider the adoption rate of a different type w'. It is given by those with signals above $k^\nu(w')$, i.e., with noise levels above $(k^\nu(w') - \theta)/\nu$. Since $\theta/\nu = k^\nu(w)/\nu - \eta$, the adoption rate of type w' is given by the fraction of type-w' agents with with noise levels above $k^\nu(w')/\nu - k^\nu/\nu + \eta$, which can be written as $\Delta k(w,w') + H_w^{-1}(1-z)$. By the LLN, such fraction is given by $1 - H_{w'}(H_w^{-1}(1-z) + \Delta k(w,w'))$. Accordingly, all the uncertainty faced by the agent can be narrowed down to her own-type average action z. In particular, the agent adjusts up or down the adoption rate of other types, compared to her own type, depending on whether they have a lower or a higher signal cutoff, respectively.

The next result establishes the dependence of individual beliefs about the average action in the population on the noise distribution whenever there is an interval $[w_j, w_{j+1}]$ of types whose signals thresholds converge in the limit as $\nu \to 0$.

Lemma 4. *If $k(w) = k_i$ for all $w \in [w_j, w_{j+1}]$ then, for each type $w \in [w_j, w_{j+1}]$ there exists a non-degenerate subset $Z_w \subset [0,1]$ and a pair of noise distributions H_w, H'_w such that $A_w(z|k;[w_j, w_{j+1}]) \neq A'_w(z|k;[w_j, w_{j+1}])$ for all $z \in Z_w$, where A_w and A'_w are the beliefs of type w about the average action in $[w_j, w_{j+1}]$ under H_w and H'_w, respectively.*

The next proposition establishes the connection between payoff separability and uniform selection.

Proposition 7. *If payoffs are separable then the GG selection is uniform. There exist a non-separable payoff function U and type distribution F such that the GG selection is not uniform.*

I finish this section by discussing the connection between global games and existing notions of risk dominance since they relate to players' individual beliefs in the game. Examples include the original definition of Harsanyi and Selten (1988), p-dominance (Morris et al., 1995), generalized risk dominance (Peski, 2010) or iterated generalized half dominance (Iijima, 2015). In contrast to ex ante RD, all these notions are defined using "interim" beliefs, that is, the beliefs of a player after learning her type. Because of their interim nature and the fact that the GG selection is equivalent to a selection based on an ex ante notion of beliefs, these versions of risk dominance are either too strong or they do not coincide with the GG selection.[11] Intuitively, these notions are based on the introduction of probabilistic beliefs that are typically independent of the payoff structure. For instance, a NE is half dominant if each player finds her equilibrium action optimal when she beliefs that her opponents choose their equilibrium actions with probability at least a half. However, as Lemma 3 shows, interim beliefs in the global game depend on the full structure of payoffs via the cutoff function and cannot be determined exogenously. In contrast, by averaging payoffs over types, the ex ante notion of risk dominance introduces ex ante beliefs with a natural interpretation that is separated from payoffs, while at the same time allowing for interim beliefs that are endogenous to the particular payoff structure of the game.

[11] For instance, it can be shown that both a generalized risk dominant equilibrium and an iterated generalized half dominant equilibrium fail to exist in some games with separable payoffs satisfying Assumption 1. It can also be shown that the classical definition of risk dominance (Harsanyi and Selten, 1988) and the GG selection lead to different equilibria in some games.

7 Conclusions

The paper studies a popular class of coordination games with strategic complementarities and proposes a new characterization of the global games equilibrium selection in terms of ex ante beliefs that have an intuitive economic meaning associated with miscoordination risk among heterogeneous agents. It also shows that a strong form of payoff symmetry is necessary for both the characterization and the robustness of the selection. The results allow for the tractable introduction of heterogeneity in many economic models and to perform comparative statics analysis, potentially enabling the use of those models in empirical and quantitative work where heterogeneity plays an important role.

A APPENDIX

A.1 Proofs of Results in Section 3

Proof of Proposition 1. Since U exhibits increasing differences, if $\Delta U(a^*, \theta, w) \geq 0$ for some w, then $\Delta U(a^*, \theta, w') > 0$ for all $w' > w$. Hence, equilibrium profile $\mathbf{a}^*$ must be monotone since otherwise some agents would be choosing 0 when their payoff difference is positive or choosing 1 when their payoff difference is negative.

Given the monotonicity of $\mathbf{a}^*$, equation (4) ensures that the lowest type choosing $a_i = 1$ (or the highest type choosing $a_i = 0$) is indifferent between the two actions. The last two conditions describe NE in which every agent type has an strict incentive to either take action 0 or action 1. □

Proof of Proposition 2. The "only if" part follows from the fact that, when $\Delta U(1 - F(w), \theta, w)$ is strictly increasing, there is a unique equilibrium since only three things can happen. First, if $\Delta U(0, \theta, \bar{w}) < 0$ then $\Delta U(1-F(w), \theta, w) < 0$ for all $w < \bar{w}$ and the unique equilibrium involves $a^* = 0$ by Proposition 1. Second, if $\Delta U(1, \theta, \underline{w}) > 0$ then $\Delta U(1 - F(w), \theta, w) > 0$ for all $w > \underline{w}$ and the unique equilibrium involves $a^* = 1$. Finally, if none of the above conditions are satisfied then there exists a unique solution w^* to $\Delta U(1-F(w^*), \theta, w^*) = 0$. To complete the argument consider the case in which $\Delta U(1 - F(w), \theta, w) = 0$ is constant in some interval of returns for some θ, leading to a continuum of equilibria. However, since $\Delta U(1 - F(w), \theta, w)$ is increasing in θ there are at most a countable number of such intervals, and thus the set of θ at which there exists multiple equilibria has Lebesgue measure zero.

Now consider the "if" part. I first focus on the case that $w' \in (\underline{w}, \bar{w})$. If $\Delta U(1 - F(w), \theta, w)$ is strictly decreasing in w at (θ', w') such that $\Delta U(1 - F(w'), \theta', w') = 0$, then the continuity of U implies that there exists a nondegenerate interval $[\theta_1, \theta_2]$ containing θ' for which the following is true. For any $\theta \in [\theta_1, \theta_2]$, (a) there exists w^* such that $\Delta U(1 - F(w^*), \theta, w^*) = 0$; (b) either there is $\hat{w} > w^*$ such that $\Delta U(1 - F(\hat{w}), \theta, \hat{w}) = 0$ or $\Delta U(1 - F(w), \theta, w) < 0$ for all $w > w^*$; and (c) either there is $\tilde{w} < w^*$ such that $\Delta U(1 - F(\tilde{w}), \theta, \tilde{w}) = 0$ or $\Delta U(1 - F(w), \theta, w) > 0$ for all $w < w^*$. Fact (a) implies there is always an equilibrium with $a^* = 1 - F(w^*)$. Fact (b) implies that there is at least another equilibrium with either $a = 1 - F(\hat{w})$ or $a = 0$. Fact (c) involves at least a third equilibrium with $a = 1 - F(\tilde{w})$ or $a = 1$.

Finally, if $w' = \underline{w}$ then (a) and (b) apply so there are at least two equilibria, while if $w' = \bar{w}$ then (a) and (c) apply and there are at least two equilibria. □

A.2 Proofs of Results in Section 4

Proof of Proposition 3. The proof logic is as follows. First, I argue that, given any $\nu > 0$ the set of equilibrium strategy profiles has a largest and a smallest element, each involving monotone (cutoff) strategies. Second, I show that there is at most one equilibrium in monotone strategies, up to differences in behavior at cutoff signals, so the least and largest equilibria are essentially the same.

Consider the game in which we fix the profile $\mathbf{s}$ of signal realizations and agents choose actions in $\{0, 1\}$ after observing their own signals. Given Assumption 1 the game satisfies the conditions of Theorem 5 in Milgrom and Roberts (1990). Accordingly, it has a smallest equilibrium $\underline{\mathbf{a}}(\mathbf{s})$ and a largest equilibrium and $\bar{\mathbf{a}}(\mathbf{s})$ such that any equilibrium profile $\mathbf{a}(\mathbf{s})$ satisfies $\underline{\mathbf{a}}(\mathbf{s}) \leq \mathbf{a}(\mathbf{s}) \leq \bar{\mathbf{a}}(\mathbf{s})$.

In addition, if we fix the actions of all agents, an agent's difference in expected payoff from choosing 0 versus 1 is increasing in $\mathbf{s}$ since the average action is kept fixed while θ is higher (in expectation) at higher signal profiles. That is, expected payoffs exhibit increasing differences w.r.t. a_i and $\mathbf{s}$, and Theorem 6 in Milgrom and Roberts (1990) applies: $\underline{\mathbf{a}}(\mathbf{s})$ and $\bar{\mathbf{a}}(\mathbf{s})$ are nondecreasing functions of $\mathbf{s}$. But, because an agent's strategy can only depend on her own signal, her strategy must be monotone on her own signal, i.e., she must be following a cutoff strategy.

To show that there is at most one equilibrium in monotone strategies, I establish the following translation result: When all cutoffs are shifted by the same amount δ, an agent's expectation of payoff differences ΔU conditional on receiving her cutoff signal strictly increases. Let $k+\delta$ denote a cutoff function shifted by δ for all w, while $\underline{k}$ and $\bar{k}$ represent the cutoffs associated with the smallest and largest equilibrium, respectively. [I omit the cutoff dependence on ν to ease notation.]

Lemma 5. *There exists $\bar{\nu} > 0$ such that, if $\nu < \bar{\nu}$ and k is a profile of equilibrium cutoffs, then $E[\Delta U(a, \theta, w)|k; k(w)] < E[\Delta U(a, \theta, w)|k + \delta; k(w) + \delta]$ for all $\delta > 0$ and all $w \in [\underline{w}, \bar{w}]$ such that $k(w) + \delta \leq \bar{k}(w)$.*

Proof. First, note that equilibrium cutoffs must lie between $\underline{\theta} - \nu/2$ and $\bar{\theta} + \nu/2$. This is because in equilibrium an agent is indifferent between both actions when she receives her signal cutoff. That is, $E[\Delta U(a, \theta, w)|k; s]$ must be zero when $s = k(w)$. Since $\Delta U(a, \theta, w) < 0$ for all $\theta < \underline{\theta}$ by Assumption 1 and $\theta \geq s - \nu/2$ it must be that $E[\Delta U(a, \theta, w)|k; s] < 0$ for all $s < \underline{\theta} - \nu/2$. A symmetric argument applies to signals above $\bar{\theta} + \nu/2$.

Given this, for all $\nu < \bar{\nu} := \min\{\underline{\theta} - \inf \Theta, \sup \Theta - \bar{\theta}\}$, the density of θ conditional on signal $s \in [\underline{\theta} - \nu/2, \bar{\theta} + \nu/2]$ is $h_w\left(\frac{s-\theta}{\nu}\right)$. Also notice that an agent of type w chooses $a_i = 0$ if she receives a signal $s < k(w)$, and thus the fraction of type-w agents choosing action 1 is given by $1 - H_w\left(\frac{k(w)-\theta}{\nu}\right)$. Hence, we can obtain the following inequality using the change of variable $\theta' = \theta + \delta$:

$$
\begin{aligned}
&E[\Delta U(a, \theta, w)|k; k(w)] = \\
&\quad \int_{k(w)-\nu/2}^{k(w)+\nu/2} \Delta U\left(1 - \int_{w'} H_{w'}\left(\frac{k(w') - \theta}{\nu}\right) dF(w'), \theta, w\right) h_w\left(\frac{k(w) - \theta}{\nu}\right) d\theta \\
&\quad < \int_{k(w)-\nu/2}^{k(w)+\nu/2} \Delta U\left(1 - \int_{w'} H_{w'}\left(\frac{k(w') - \theta}{\nu}\right) dF(w'), \theta + \delta, w\right) h_w\left(\frac{k(w) - \theta}{\nu}\right) d\theta \\
&\quad = \int_{k(w)+\delta-\nu/2}^{k(w)+\delta+\nu/2} \Delta U\left(1 - \int_{w'} H_{w'}\left(\frac{k(w') + \delta - \theta'}{\nu}\right) dF(w'), \theta', w\right) h_w\left(\frac{k(w) + \delta - \theta'}{\nu}\right) d\theta' \\
&\quad = E[\Delta U(a, \theta, w)|k + \delta; k(w) + \delta].
\end{aligned}
$$

□

I finish the proof by arguing that $\underline{k}(w) = \bar{k}(w)$ for all w. Assume, by way of contradiction, that $\underline{k}(w) < \bar{k}(w)$ for some w. Denote $\hat{w} = \arg\max_w(\bar{k}(w) - \underline{k}(w))$

the type with the biggest difference in signal cutoffs between the largest and the smallest equilibria. Also, let $\hat{\delta} = \bar{k}(\hat{w}) - \underline{k}(\hat{w})$. By Lemma 5, we have that

$$0 = E[\Delta U(a, \theta, w)|\underline{k}; \underline{k}(\hat{w})] < E[\Delta U(a, \theta, w)|\underline{k}+\hat{\delta}; \bar{k}(\hat{w})] \leq E[\Delta U(a, \theta, w)|\bar{k}; \bar{k}(\hat{w})] = 0,$$

where the last inequality comes from the fact that a is higher at at $\bar{k}$ than at $\underline{k}+\hat{\delta} \geq \bar{k}$, so the expected payoff difference of $\hat{w}$ conditional on $x = \bar{k}(\hat{w})$ is higher. □

A.3 Proofs of Results in Subsection 5.2

Proof of Lemma 1. If $\hat{w} \in (\underline{w}, \bar{w})$ is not a NE then $\Delta\tilde{U}(1 - F(\hat{w}), \theta, \hat{w}) \neq 0$ by Proposition 1. Given the continuity of $\Delta\tilde{U}(1-F(\hat{w}), \theta, \hat{w})$ we can increase or decrease $\hat{w}$ so that the objective function in (10) goes up. If $\hat{w} = \underline{w}$ then it must be that $\Delta\tilde{U}(1, \theta, \hat{w}) < 0$ so increasing $\hat{w}$ increases the value of the objective function. A symmetric argument applies to the case $\hat{w} = \bar{w}$.

Condition (ii) directly follows from the change of variable $a = 1 - F(w)$. Condition (iii) follows from the fact that the difference in the value of the objective function (10) between NE w^* and $w^{*'}$ is given by (12). Hence, if this difference is positive for all alternative NE in $W^*(\theta)$ then w^* must be ex ante RD. We obtain (iv) by substituting $\Delta\tilde{U}(1 - F(w), \theta, w) = u(1 - F(w), \theta) + v(\theta, w)$ into (12) and applying a change of variable $a = 1 - F(w)$ to $\int_{w^*}^{w^{*'}} u(1 - F(w), \theta)dF(w)$. □

Proof of Proposition 4. I first prove existence. $W^*(\theta)$ is non-empty since the game is supermodular so it must have a least and largest NE, both in pure strategies (Milgrom and Roberts, 1990). $W^*(\theta)$ is also compact given that the set of player types is bounded and that the limit of any convergent sequence of types in $W^*(\theta)$ must also be the marginal type of a NE, i.e., must belong to $W^*(\theta)$. The latter is due to the continuity of payoffs, which implies that the sequence of payoff differences associated with the sequence of types also converges and thus condition (4) must be satisfied by the limit type. By Lemma 1, finding the ex ante RD equilibrium involves finding the marginal type $w^* \in W^*(\theta)$ that solves (10). Since the objective function is continuous and $W^*(\theta)$ is non-empty and compact, by the extreme value theorem the set of maximizers is non-empty, that is, an ex ante RD exists.

I next prove that the ex ante RD is essentially unique. I do so by showing that, if there are more than one ex ante RD equilibria at some θ, then there exist $\theta' < \theta$ and

$\theta'' > \theta$ such that there is only one ex ante RD equilibrium for parameters in $(\theta', \theta) \cup (\theta, \theta'')$. Since Θ can only be partitioned in a countable number of non-degenerate intervals then the set of θ at which there are multiple ex ante RD equilibria must be countable, i.e., must have Lebesgue measure zero.

First, note that a NE w^* such that $\Delta\tilde{U}(1 - F(w^*), \theta, w^*) = 0$ with $\Delta\tilde{U}(1 - F(w), \theta, w)$ strictly decreasing in w at w^* cannot be ex ante RD. The reason is that, by the continuity of $\Delta\tilde{U}$ w.r.t. w and Proposition 1, there must be another NE $w^{*'}$ such that, either $w^{*'} < w^*$ and $\tilde{U}(1 - F(w), \theta, w) > 0$ for all $w \in (w^{*'}, w^*)$, or $w^{*'} > w^*$ and $\tilde{U}(1 - F(w), \theta, w) < 0$ for all $w \in (w^*, w^{*'})$. Hence, w^* violates condition (iii) in Lemma 1.[12] Accordingly, there are only three candidates for ex ante RD equilibria: (a) NE w^* with $\Delta\tilde{U}(1 - F(w^*), \theta, w^*) = 0$ such that $\Delta\tilde{U}(1 - F(w), \theta, w)$ is increasing at w^*; (b) $\underline{w}$ with $\Delta\tilde{U}(1, \theta, \underline{w}) \geq 0$; and (c) $\bar{w}$ with $\Delta\tilde{U}(0, \theta, \bar{w}) \leq 0$.

Second, note that if there exist two (or more) maximizers of (10) for a given θ, condition (iii) in Lemma 1 implies that, for any pair of maximizers w^* and $w^{*'}$,

$$\int_{w^*}^{w^{*'}} \Delta\tilde{U}(1 - F(w), \theta, w) dF(w) = 0, \tag{26}$$

otherwise one of them would yield a higher value of the objective function in (10). Given this, consider an infinitesimal increase in θ. Since $\Delta\tilde{U}(1-F(w), \theta, w)$ is strictly increasing in θ, the integrand in (26) goes up as θ increases. In addition, both integration limits (weakly) go down. But this means that if $w^* < w^{*'}$ and both limits change continuously with θ, the LHS of (26) strictly increases. The reason is that, given that w^* is ex ante RD, either $\Delta\tilde{U}(1 - F(w), \theta, w)$ is equal to zero and increasing at w^* or $w^* = \underline{w}$ and $\Delta\tilde{U}(1, \theta, \underline{w}) \geq 0$. Hence, the integrand must be positive for values of w just above w^*. Similarly, $\Delta\tilde{U}(1 - F(w), \theta, w)$ is equal to zero and increasing at $w^{*'}$ or $w^* = \bar{w}$ and $\Delta\tilde{U}(0, \theta, \bar{w}) \leq 0$ so the integrand must be negative for values just below $w^{*'}$. Figure 4 illustrates the increase in the LHS, represented by the shaded area, associated with an increase in θ in the context of Example 1. Similarly, an infinitesimal drop in θ leads to a drop in the LHS of (26). Accordingly, there is only one ex ante RD equilibrium in an open neighborhood $(\theta', \theta) \cup (\theta, \theta'')$. Finally, notice that, by the continuity of $\Delta\tilde{U}$, the only way one of the integration limits might not change continuously is because one of them ceases

[12] This is illustrated in Figure 2, where switching from w_2^* to either w_1^* or w_2^* increases the value of the objective function by the size of areas A and B, respectively.

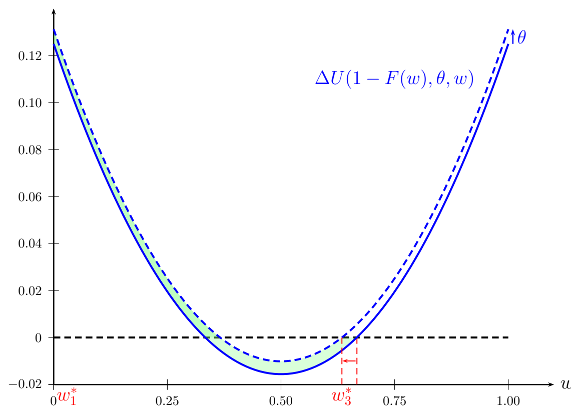

Figure 4: Effect of an increase in θ

to be a NE and thus only one of them can be ex ante RD. □

Proof of Proposition 5 and Corollary 1. The function w_{RD} is well-defined since an ex ante RD equilibrium exists for all $\theta \in \Theta$ by Proposition 4.

I next argue that w_{RD} is decreasing. From the proof of Proposition 4 we know that $\Delta\tilde{U}(1 - F(w), \theta, w)$ is increasing in w at an ex ante RD equilibrium w^*_{RD} that satisfies $\Delta\tilde{U}(1-F(w^*_{RD}), \theta, w^*_{RD}) = 0$. Given this, consider two cases, when there is a unique ex ante RD equilibrium and when there are two or more. If there is a unique ex ante RD equilibrium w^*_{RD} with $\Delta\tilde{U}(1-F(w^*_{RD}), \theta, w^*_{RD}) = 0$ then a small increase in θ leads to an increase in $\Delta\tilde{U}(1 - F(w^*_{RD}), \theta, w^*_{RD})$. Since $\Delta\tilde{U}(1 - F(w), \theta, w)$ is increasing in w then w_{RD} must go down to satisfy the NE conditions in Proposition 1. If $\Delta\tilde{U}(1 - F(w^*_{RD}), \theta, w^*_{RD}) \neq 0$ then it must be that $w^*_{RD} \in \{\underline{w}, \bar{w}\}$. In such a case, a small increase in θ does not affect w^*_{RD} since both $\Delta\tilde{U}(1 - F(w^*_{RD}), \theta, w^*_{RD})$ and the objective function in the maximization problem (10) are continuous in θ.

Next consider the case in which there are multiple ex ante RD equilibria at θ. The proof of Proposition 4 shows that this can only happen in isolated points of Θ. Moreover, I argue in that proof that these equilibria satisfy (26), whose left hand side is strictly increasing in θ. Accordingly, since w_{RD} selects the lowest of them by definition, an infinitesimal increase in θ must lead to a drop in the ex ante RD from the highest to the lowest ex ante RD marginal type at θ. This also proves that w_{RD} is right-continuous, and that there is a countable collection $\theta_1 < \theta_2 < \cdots < \theta_n$ satisfying properties 1 and 2 in the proposition.

Property 3 is a consequence of w_{RD} being decreasing: if (17) holds then $\underline{w}$ is an ex ante RD equilibrium. Since it is the lowest one, w_{RD} must select it by definition.

Property 4 states that $\bar{w}$ is selected by w_{RD} whenever it is the unique ex ante RD. This happens either because it is the unique NE, i.e., $\Delta U(1-F(w), \theta, w) < 0$ for all $w < \bar{w}$, or because the other NE lead to a lower value of the objective function in the maximization problem (10), i.e., when (18) is satisfied.

Corollary 1 directly follows from conditions (13) and (16). □

Proof of Corollary 2. It is straightforward to verify from the argument in the proof of the "if" part in Proposition 2 that if $\Delta\tilde{U}(1-F(w), \theta, w)$ is decreasing or strictly quasiconcave in w then there is a non-degenerate interval of θ such that there are exactly three equilibria, with one of them having a marginal type w^* such that $\Delta\tilde{U}(1-F(w), \theta, w)$ is equal to zero and decreasing at w^*. The proof of Proposition 4 shows that the latter cannot be ex ante RD, implying that only the lowest and the highest adoption equilibria of the three can be ex ante RD. Finally, the no adoption equilibrium is the unique NE at $\theta < \underline{\theta}$ while the full adoption is the unique one for $\theta > \bar{\theta}$. Hence, since $\Delta\tilde{U}(1-F(w), \theta, w)$ is increasing and continuous in θ there must be a unique switch from the low adoption to the high adoption equilibrium, leading to a single discontinuity in w_{RD}. □

A.4 Proofs of Results in Subsection 5.3

Proof of Proposition 6 and Corollary 3. As shown in the proof of Proposition 3, equilibrium cutoffs are bounded. Hence, any subsequence of the sequence of cutoff functions $\{k^\nu\}$ has a convergent subsequence. Moreover, since $[\underline{w}, \bar{w}]$ is compact, the convergence is uniform. Accordingly, I need to show that the limit of any convergent subsequence is given by the cutoff function $k(w)$ satisfying the proposition.

Recall that equilibrium cutoffs satisfy the system of indifference conditions (6), which under payoff separability can be written as

$$E[\Delta\tilde{U}(a, \theta, w)|k^\nu; s = k^\nu(w)] = 0 \text{ for all } w \in [\underline{w}, \bar{w}].$$

Also note that, since $\theta \in [s - \nu/2, s + \nu/2]$ and payoff differences are increasing in

θ, the LHS of the above indifference conditions satisfies

$$E[\Delta\tilde{U}(a, k^\nu(w) - \nu/2, w)|k^\nu; s = k^\nu(w)]$$
$$\leq E[\Delta\tilde{U}(a, \theta, w)|k^\nu; s = k^\nu(w)]$$
$$\leq E[\Delta U(a, k^\nu(w) + \nu/2, w)|k^\nu; s = k^\nu(w)].$$

Accordingly, the (Lipschitz) continuity of $\Delta\tilde{U}$ implies that the limit cutoff function k of any convergent subsequence of $\{k^\nu\}$ satisfies the system of indifference conditions

$$\lim_{\nu\to 0} E[\Delta\tilde{U}(a, k(w), w)|k^\nu; s = k^\nu(w)] = 0 \text{ for all } w \in [\underline{w}, \bar{w}]. \tag{27}$$

That is,

$$\lim_{\nu\to 0} \int_0^1 \Delta\tilde{U}(a, k(w), w) dA_w(z|k^\nu) = 0 \text{ for all } w \in [\underline{w}, \bar{w}], \tag{28}$$

where $A_w(a|k^\nu)$ is the cdf representing the beliefs of an agent of type w receiving signal $s = k^\nu(w)$ when players use equilibrium cutoffs k^ν.

I next argue that the limit cutoff function must be decreasing. To see why assume, by way of contradiction, that there exist two types w and w' such that $w > w'$ and $k(w) > k(w')$. Since the support of signals collapses into the actual value of θ as $\nu \to 0$, an agent receiving $s = k(w)$ knows that the average action must be higher than when she receives signal $s' = k(w')$ regardless of her type, given that other players' signals are strictly higher when $s = k(w) > s'$. But then, since $\Delta\tilde{U}$ is increasing in all its arguments, we must arrive to the following contradiction:

$$0 = \lim_{\nu\to 0} E[\Delta\tilde{U}(a, k(w), w)|k^\nu; s = k^\nu(w)] > \lim_{\nu\to 0} E[\Delta\tilde{U}(a, k(w'), w')|k^\nu; s = k^\nu(w')] = 0.$$

Given that k is decreasing, we can partition of the set of types into two types of intervals: intervals at which the limit cutoff $k(w)$ is strictly decreasing and intervals of types that follow the same limit cutoff.

For a type w at which $k(w)$ is strictly decreasing, their limit belief places all the probability mass on the actual average action, given by $1 - F(w)$. This is because, since all the signals are within ν of $s = k(w)$ and types lower than w have higher signal cutoffs their signals must be below their respective cutoffs, while higher types receive signals above their cutoffs. Hence, (28) translates into

$$\Delta\tilde{U}(1 - F(w), k(w), w) = 0. \tag{29}$$

If a type w belongs to an interval of types (w', w'') at which $k(w) = \hat{k}$ is constant, she is certain that the average action falls within $1 - F(w'')$ and $1 - F(w')$ but still deems the average action of types in (w', w'') as random. Let $A_w(z|k^\nu; (w', w''))$ be the belief of type w about the average action z of types in (w', w'') conditional on receiving $s = k^\nu(w)$. We can then write her limit indifference condition (28) as

$$\lim_{\nu\to 0} \int_0^1 \Delta\tilde{U}(1 - F(w'') + z(F(w'') - F(w')), k(w), w) dA_w(z|k^\nu; (w', w'')) = 0.$$

Next, we can average the limit indifference conditions of types in (w', w'') and obtain

$$\int_{w'}^{w''} \lim_{\nu\to 0} \int_0^1 \Delta\tilde{U}(1-F(w'')+z(F(w'')-F(w')), \hat{k}, w) dA_w(z|k^\nu; (w', w'')) dF(w|w \in (w', w'')) = 0.$$

Substituting for $\Delta\tilde{U}$ we can express this average indifference condition as

$$\int_{w'}^{w''} \lim_{\nu\to 0} \int_0^1 u(1 - F(w'') + z(F(w'') - F(w')), \hat{k}) dA_w(z|k^\nu; (w', w'')) dF(w|w \in (w', w''))$$
$$+ \int_{w'}^{w''} v(\hat{k}, w) dF(w|w \in (w', w'')) = 0. \tag{30}$$

Finally, recall that the proof of Proposition 3 establishes that $k^\nu(w) \in [\underline{\theta} - \nu/2, \bar{\theta} + \nu/2]$, implying that $k^\nu(w) \in (\inf\Theta + \nu/2, \sup\Theta - \nu/2)$ for all $\nu < \bar{\nu}$. Hence, as Lemma 2 establishes, agent beliefs satisfy the belief constraint. That is,

$$\lim_{\nu\to 0} \int_{w'}^{w''} A_w(z|k^\nu; (w', w'')) dF(w|w \in (w', w'')) = z \text{ for all } z \in [0, 1], \tag{31}$$

which I use to substitute for the average limit belief in (30) to obtain

$$\int_0^1 u(1 - F(w'') + z(F(w'') - F(w')), \hat{k}) dz + \int_{w'}^{w''} v(\hat{k}, w) dF(w|w \in (w', w'')) = 0.$$

Applying the change of variable $a = 1 - F(w'') + z(F(w'') - F(w'))$ and noting that

$f(w) = \frac{f(w|w\in(w',w''))}{F(w'')-F(w')}$ yields

$$\int_{1-F(w'')}^{1-F(w')} u(a,\hat{k})da + \int_{w'}^{w''} v(\hat{k},w)dF(w) = 0. \tag{32}$$

I finish the proof by arguing that there can only be a unique partition of types such that limit cutoffs satisfy (29) at w when $k(w)$ is strictly decreasing and (32) at any interval (w', w'') where $k(w)$ is constant. To do so I establish that they are equivalent to the conditions in Proposition 5 characterizing the unique function w_{RD} governing the ex ante RD selection. Such an equivalence also proves Corollary 3.

Define the mapping $k_{RD} : w \to \Theta$ as

$$k_{RD}(w) = \begin{cases} \min\{\theta : w_{RD}(\theta) = w\} & w = \underline{w} \\ \theta \text{ s.t. } w_{RD}(\theta) = w & w \in (\underline{w}, \bar{w}) \setminus \bigcup_j [w_{RD}(\theta_j), w^+_{RD}(\theta_j)] \\ \theta_j & w \in [w_{RD}(\theta_j), w^+_{RD}(\theta_j)] \\ \max\{\theta : w_{RD}(\theta) = w\} & w = \bar{w}, \end{cases}$$

where $\{\theta\}_j$ is the collection of parameters satisfying (16) in Proposition 5.

It is straightforward to check that k_{RD} is well-defined, continuous and decreasing. In addition, since w_{RD} is continuous in $\Theta \setminus \{\theta_j\}_{j=1}^J$, k_{RD} must be strictly decreasing except at θ such that $w_{RD} \in \{\underline{w}, \bar{w}\}$. This implies that k_{RD} is strictly decreasing in $(\underline{w}, \bar{w}) \setminus \bigcup_j [w_{RD}(\theta_j), w^+_{RD}(\theta_j)]$. Moreover, since $w_{RD}(\theta)$ is a NE Proposition 1 implies that, whenever $w_{RD}(\theta)$ is strictly decreasing, it satisfies

$$\Delta U(1 - F(w_{RD}(\theta)), w_{RD}(\theta), w_{RD}(\theta)) = 0,$$

or, equivalently, k_{RD} satisfies condition (29) for all $(\underline{w}, \bar{w}) \setminus \bigcup_j [w_{RD}(\theta_j), w^+_{RD}(\theta_j)]$.

Next, note that Lemma 1 implies that (16) can be written as

$$\int_{1-F(w^+_{RD}(\theta_j))}^{1-F(w_{RD}(\theta_j))} u(a,\theta_j)da + \int_{w_{RD}(\theta_j)}^{w^+_{RD}(\theta_j)} v(\theta_j,w)dF(w) = 0,$$

which is equivalent to condition (32).

Finally, since $k_{RD}(\underline{w})$ is the lowest θ such that $\underline{w}$ is the ex-ante RD marginal type, property 3 in Proposition 5 implies that $\Delta\tilde{U}(1 - F(\underline{w}), k_{RD}(\underline{w}), \underline{w}) \geq 0$. Similarly,

$k_{RD}(\bar{w})$ being the highest θ at which $\bar{w}$ is the ex-ante RD marginal type implies that $\Delta\tilde{U}(1-F(\bar{w}), k_{RD}(\bar{w}), \bar{w}) \leq 0$.

Propositions 4 and 5 guarantee that there exists a unique function k_{RD} satisfying all these conditions. But since these conditions coincide with the conditions in Proposition 6, they imply the existence of a unique signal cutoff function k in the limit equilibrium of the global game, characterized by Proposition 6. □

Proof of Lemma 2. (adapted from Drozd and Serrano-Padial (2018) and Sakovics and Steiner, 2012)[13]

I prove Lemma 2 through a series of steps. Fix a cutoff function κ mapping types to signals in $[\inf \Theta + \nu/2, \sup \Theta - \nu/2]$, and a measurable subset of types W'.

First, I define "virtual signals" $\tilde{s} = s - \kappa(w)$ for all $w \in W'$, which exhibit a common default threshold $\tilde{\kappa} = 0$. Also define the 'extended type' of a player as the signal-type tuple (s, w).

Second, I show that the uniform prior assumption implies that the density associated with signal $s = \kappa(w)$ for a player of type (s, w), conditional on $\tilde{s} = 0$ and on $w \in W'$, is given by the conditional density of types w in subset W', i.e.,

$$Pr(\kappa(w), w|\tilde{s} = 0, W') = \frac{f(w)}{\int_{W'} f(w)dw}, \tag{33}$$

where $Pr(s, w|\cdot)$ denotes the conditional probability density of extended type (s, w).

Third, I show that the average action of types in subset W', denoted by $a(\theta, W')$, is uniformly distributed in $[0, 1]$ conditional on $\tilde{s} = 0$. That is,

$$Pr(a(\theta, W') < z|\tilde{s} = 0, W') = z. \tag{34}$$

Equipped with (33) and (34), we obtain the belief constraint (31):

$$\begin{aligned} z &= Pr(a(\theta, W') < z|\tilde{s} = 0, W') \\ &= \int_{W'} Pr\left(a(\theta, W') < z\middle| s = \kappa(w), W'\right) Pr(s = \kappa(w)|\tilde{s} = 0, W')dw \\ &= \frac{1}{\int_{W'} f(w)dw} \int_{W'} Pr\left(a(\theta, W') < z\middle| s = \kappa(w), W'\right) f(w)dw, \end{aligned}$$

[13] It adapts Lemma 7 in Drozd and Serrano-Padial (2018) to a continuous type distribution. Their result generalizes the belief constraint in Sakovics and Steiner (2012) to any subset of types.

where $Pr\left(a(\theta, W') < z \middle| s = \kappa(w), W'\right)$ represents the belief $A_w(z|\kappa; W')$ of type w.

I prove (33) by pinning down the marginal distributions of (s, w) and $\tilde{s}$. Given the above restriction on cutoffs κ, I focus only on signals $s \in [\inf \Theta + \nu/2, \sup \Theta - \nu/2]$. Recall that θ is uniformly distributed and independent of η and w. Accordingly, the joint density of (s, w, θ) is

$$\begin{aligned} Pr(s, w, \theta|W') &= Pr(s|w, \theta, W')Pr(w|\theta, W')Pr(\theta|W') = Pr(s|w, \theta)Pr(w|W')Pr(\theta) \\ &= \left(h_w\left(\frac{s-\theta}{\nu}\right)\frac{1}{\nu}\right)\left(\frac{f(w)}{\int_{W'} f(w)dw}\right)\left(\frac{1}{\sup \Theta - \inf \Theta}\right). \end{aligned}$$

We obtain the marginal density of (s, w) by integrating the above expression:

$$\begin{aligned} Pr(s, w|W') &= \int_{s-\nu/2}^{s+\nu/2} Pr(s, w, \theta|W')d\theta = \int_{s-\nu/2}^{s+\nu/2} h\left(\frac{s-\theta}{\nu}\right)\frac{1}{\nu}\frac{f(w)}{\int_{W'} f(w)dw}\frac{1}{\sup \Theta - \inf \Theta}d\theta \\ &= \frac{f(w)}{\int_{W'} f(w)dw}\frac{1}{\sup \Theta - \inf \Theta}. \end{aligned}$$

The marginal density of the virtual signal $\tilde{s} = s - \kappa(w)$ is given by

$$Pr(s = \tilde{s} + k(w)|W') = \int_{W'} Pr(\tilde{s} + \kappa(w), w|W')dw = \frac{1}{\sup \Theta - \inf \Theta},$$

for all $\tilde{s}$ such that $\tilde{s} + \kappa(w) \in [\inf \Theta + \nu/2, \sup \Theta - \nu/2]$. Since $\tilde{s} = 0$ satisfies this condition given the above bounds on $\kappa(w)$, we have that

$$Pr(k(w), w|\tilde{s} = 0, W') = \frac{Pr(k(w), w|W')}{Pr(s = k(w)|W')} = \frac{f(w)}{\int_{W'} f(w)dw}.$$

To prove (34) first note that the virtual noise $\tilde{\eta} = (\tilde{s} - \theta)/\nu$ follows the mixture distribution $\left\{H_w\left(\tilde{\eta} + \frac{\kappa(w)}{\nu}\right), \frac{f(w)}{\int_{W'} f(w)dw}\right\}_{w \in W'}$. This implies that the virtual noise belongs to type w with probability $\frac{f(w)}{\int_{W'} f(w)dw}$. In addition, its distribution conditional on type w is given by the noise distribution evaluated at $\eta = \tilde{\eta} + \kappa(w)/\nu$. But note that the mixture distribution does not depend on θ so the random variable $\tilde{\eta}$ is i.i.d. across agents and independent of θ. Let G be the cdf of $\tilde{\eta}$ and define $G^{-1}(z) = \inf\{\tilde{\eta} : G(\tilde{\eta}) = z\}$. Given the definition of virtual noise, the average action in subset W' is given by the fraction of agents in W' whose virtual signal is lower than zero, i.e., by the cdf of the virtual noise G evaluated at $-\theta/\nu$. This yields expression (34) given that

$$Pr(a(\theta, W') < z|\tilde{s} = 0, W') = Pr(G(-\theta/\nu) < z|\tilde{s} = 0, W') = Pr(G(\tilde{\eta}) < z)$$
$$= Pr(\tilde{\eta} < G^{-1}(z)) = G(G^{-1}(z)) = z. \qquad \square$$

A.5 Proofs of Results in Section 6

Proof of Lemma 3. Consider an agent of type w with signal $s = \theta + \nu\eta = k^\nu(w)$. The mass of agents with the same type choosing $a_i = 1$ are those with signals $s \geq k^\nu(w)$, i.e., those with signal noise above η. By the exact LLN such mass is given by $1 - H_w(\eta)$. Since the agent does not observe η and it is i.i.d. she deems $H_w(\eta)$ as a random variable uniformly distributed in $[0, 1]$. Accordingly $a_w = 1 - H_w(\eta) \sim U[0, 1]$. This proves the first part of the Lemma.

Next, consider the agent's beliefs about the fraction of agents of a type $w' \neq w$ choosing $a_i = 1$. It is given by the agents of type w' receiving signals $s' = \theta + \nu\eta' \geq k^\nu(w')$. Since $\theta + \nu\eta = k^\nu(w)$ this condition can be expressed as follows

$$\theta + \nu\eta' - (\theta + \nu\eta) \geq k^\nu(w') - k^\nu(w) \Rightarrow \eta' \geq \eta + \Delta k(w, w').$$

By the exact LLN the mass of agents with signal noise satisfying this condition is given by $1 - H_{w'}(\eta + \Delta k(w, w'))$. Let z denote the mass of agents of type w choosing $a = 1$ when $s = k(w)$. By the above argument $z = 1 - H_w(\eta)$ or $\eta = H_w^{-1}(1 - z)$. Hence, $a_{w'} = 1 - H_{w'}(H_w^{-1}(1 - z) + \Delta k(w, w'))$. To complete the proofs of parts 2 and 3 notice that types $w' < w$ have a weaker incentive to choose $a_i = 1$ and thus their threshold is higher and their average action is lower. Accordingly, no one adopts, i.e., $H_{w'}(H_w^{-1}(1 - z) + \Delta k(w, w')) = 1$ for all z satisfying $H_w^{-1}(1 - z) + \Delta k(w, w') = H_{w'}^{-1}(1) = \frac{1}{2}$, which yields the expression of the average action in part 2. A symmetric argument applies to the case of $w' > w$. $\square$

Proof of Lemma 4. The proof uses the characterization of individual beliefs about the average action of a single type to construct the individual beliefs A_w of a type about the average action in $[w_j, w_{j+1}]$ as a function of own-type average action, which is uniformly distributed by Lemma 3. Then, I apply this characterization to the family of power noise distributions to show that $A_w(a|k, [w_j, w_{j+1}])$ differs across noise distributions in a nondegenerate interval of z.

First, notice that, by Lemma 3, the belief about own-type average action is independent of the noise distribution since it is the uniform distribution on the unit interval. Accordingly, we can express the belief of type w about average action of types in the interval $[w_j, w_{j+1}]$, conditional on $s = k(w)$, as a function of z as follows

$$a(z, [w_j, w_{j+1}]) = \int_{w_j}^{w_{j+1}} a_{w'}(z) dF_j(w'), \; z \sim U[0,1], \tag{35}$$

where $F_j(w) := F(w|w \in [w_j, w_{j+1}])$. To show that the conditional distribution of the average action changes with the noise distribution, it suffices to find a pair of noise distribution families for which $a(z, [w_j, w_{j+1}])$ is different under H_w than under H'_w in a non-degenerate interval of z. Using Lemma 3, we have that

$$\begin{aligned} a(z, [w_j, w_{j+1}]) &= \int_{w_j}^{w_{j+1}} a_{w'}(z) dF_j(w') \\ &= \int_{w_l(z)}^{w_h(z)} \left(1 - H_{w'}(H_w^{-1}(1-z) + \Delta k(w, w'))\right) dF_j(w') + 1 - F_j(w_h(z)) \\ &= 1 - F_j(w_l(z)) - \int_{w_l(z)}^{w_h(z)} H_{w'}(H_w^{-1}(1-z) + \Delta k(w, w')) dF_j(w') \end{aligned} \tag{36}$$

where $w_l(z)$ and $w_h(z)$ satisfy $z = 1 - H_w(\frac{1}{2} - \Delta k(w, w_l))$ and $z = 1 - H_w(-\frac{1}{2} - \Delta k(w, w_h))$, respectively. Since $a(z, [w_j, w_{j+1}])$ is increasing in z we can differentiate the above expression to get

$$\begin{aligned} \frac{da(z, [w_j, w_{j+1}])}{dz} &= -f_j(w_l(z)) \frac{dw_l(z)}{dz} + H_{w'}(H_w^{-1}(1-z) + \Delta k(w, w_l(z))) f_j(w_l(z)) \frac{dw_l(z)}{dz} \\ &- \int_{w_l(z)}^{w_h(z)} \frac{d}{dz} H_{w'}(H_w^{-1}(1-z) + \Delta k(w, w')) dF_j(w') \\ &- H_{w'}(H_w^{-1}(1-z) + \Delta k(w, w_h(z))) f_j(w_h(z)) \frac{dw_h(z)}{dz} \\ &= - \int_{w_l(z)}^{w_h(z)} \frac{d}{dz} H_{w'}(H_w^{-1}(1-z) + \Delta k(w, w')) dF_j(w'). \end{aligned}$$

Next, consider the family of power distributions $H_w(\eta) = (\eta + 1/2)^\alpha$ for all w, with

$\alpha > 0$. Under these distributions we have that, for any $w' \in [w_l(z), w_h(z)]$,

$$H_{w'}(H_w^{-1}(1-z) + \Delta k(w,w')) = ((1-z)^{1/\alpha} + \Delta k(w,w'))^{\alpha}.$$

Hence, the above derivative becomes

$$\frac{da(z,[w_j,w_{j+1}])}{dz} = \int_{w_l(z)}^{w_h(z)} (1-z)^{\frac{1-\alpha}{\alpha}} \left((1-z)^{1/\alpha} + \Delta k(w,w')\right)^{\alpha-1} dF_j(w'). \quad (37)$$

Finally, to show how $a(z,[w_j,w_{j+1}])$ depends on H one can compare its derivative for the case of $\alpha = 1$ to the case of $\alpha < 1$, evaluated at at $z = 1$. In the former case we have

$$\left.\frac{da(z,[w_j,w_{j+1}])}{dz}\right|_{z=1} = \int_{w_l(z)}^{w_h(z)} dF_j(w') = F_j(w_h(1)) - F_j(w_l(1)).$$

The right hand side in this expression is strictly positive for any type in the interior of $[w_j, w_{j+1}]$ since $1 = 1 - H_w(1/2 - \Delta k(w, w_l(0)))$ implies that $\Delta k(w, w_l(1)) = 1$, and $1 = 1 - H_w(-1/2 - \Delta k(w, w_h(1)))$ implies that $\Delta k(w, w_h(1)) = 0$. Accordingly, it must be that $w_l(1) < w_h(1)$ given that $\Delta k(w, w')$ is strictly decreasing in w' due to the monotonicity of payoffs w.r.t. types.

In contrast, when $\alpha < 1$, $\left.\frac{da(z,[w_j,w_{j+1}])}{dz}\right|_{z=1} = 0$. Given the continuity of $a(\cdot,[w_j,w_{j+1}])$, this implies that there exists an interval $[\underline{z}, 1]$ such that $a(z,[w_j,w_{j+1}])$ is strictly higher or strictly lower for all $z \in (\underline{z}, 1]$ under $\alpha = 1$ than under $\alpha < 1$.[14] □

Proof of Proposition 7. Corollary 3 implies that payoff separability leads to uniform selection given that the GG selection coincides with the ex ante RD equilibrium, which does not depend on H_w. To show that uniform selection can fail under non-separability I introduce an example with a two-type distribution and argue that the selection is not uniform for any continuous F arbitrarily close to it.

Consider payoffs $U(0,a,\theta,w) = 1$ and $U(1,a,\theta,w) = \theta + 2wa + (1-w)a^2$, which lead to

$$\Delta U(a,\theta,w) = \theta - 1 + 2wa + (1-w)a^2. \quad (38)$$

[14] If $a(0,[w_j,w_{j+1}])$ is equal in both cases then it is strictly higher in the interval under $\alpha = 0$ since the derivative is strictly positive. If $a(0,[w_j,w_{j+1}])$ differs across α then such an interval exists given the continuity of $a(\cdot,[w_j,w_{j+1}])$.

In addition, let F place all the mass on $\{0,1\}$, with $F(0) = Prob(w = 0) = 1/2$. It is straightforward to check that for $\theta \in [0,1]$ both $a^* = 0$ and $a^* = 1$ are NE, while there is a third (non-stable) equilibrium in which only $w = 1$ adopt for $\theta \in [0, 3/4]$. Hence, the GG selection involves both types using the same limit cutoff k.

For any noise level $\nu > 0$ let $\Delta = \frac{k^\nu(0) - k^\nu(1)}{\nu} > 0$. Next consider the individual beliefs about the average action given by (35) under the family of power distributions. For the low type $w = 0$, $w_l(z) = 0$ for all z, while $w_h(z) = \begin{cases} 1 & z < 1 - \Delta^\alpha \\ 0 & z \geq 1 - \Delta^\alpha \end{cases}$. Hence, her beliefs about the average action, denoted by $a^0(z)$, are

$$a^0(z) = \begin{cases} \frac{z}{2} + \frac{1}{2}\left(1 - ((1-z)^{1/\alpha} - \Delta)^\alpha\right) & z < 1 - \Delta^\alpha \\ \frac{z}{2} + \frac{1}{2} & z \geq 1 - \Delta^\alpha. \end{cases} \tag{39}$$

Similarly, for the high type $w = 1$, $w_h(z) = 1$ for all z, while $w_l(z) = \begin{cases} 1 & z < 1 - (1-\Delta)^\alpha \\ 0 & z \geq 1 - (1-\Delta)^\alpha \end{cases}$. This leads to beliefs

$$a^1(z) = \begin{cases} \frac{z}{2} & z < 1 - (1-\Delta)^\alpha \\ \frac{z}{2} + \frac{1}{2}\left(1 - ((1-z)^{1/\alpha} + \Delta)^\alpha\right) & z \geq 1 - (1-\Delta)^\alpha. \end{cases} \tag{40}$$

Since $s \to \theta$ as $\nu \to 0$, the indifference conditions pinning down k are

$$0 = \int_0^1 \Delta U(a^w(z), k, w)dz, \quad w = 0, 1.$$

Accordingly, k and Δ solve the following system of two equations

$$0 = k - 1 + \int_0^{1-\Delta^\alpha} \left(\frac{z}{2} + \frac{1}{2}\left(1 - \left((1-z)^{1/\alpha} - \Delta\right)^\alpha\right)\right)^2 dz + \int_{1-\Delta^\alpha}^1 \left(\frac{z}{2} + \frac{1}{2}\right)^2 dz,$$

$$0 = k - 1 + 2\int_0^{1-(1-\Delta)^\alpha} \frac{z}{2}dz + 2\int_{1-(1-\Delta)^\alpha}^1 \left(\frac{z}{2} + \frac{1}{2}\left(1 - \left((1-z)^{1/\alpha} + \Delta\right)^\alpha\right)\right) dz.$$

To show that the GG selection is not uniform I solve these conditions for two different values of α, namely, $\alpha = 1$ (the uniform distribution) and $\alpha = 2$ and show that k differs across the two solutions.

When $\alpha = 1$ indifference conditions are

$$0 = k - 1 + \int_0^{1-\Delta} \left(z + \frac{\Delta}{2}\right)^2 dz + \int_{1-\Delta}^{1} \left(\frac{z}{2} + \frac{1}{2}\right)^2 dz,$$

$$0 = k - 1 + 2\int_0^{\Delta} \frac{z}{2} dz + 2\int_{\Delta}^{1} \left(z - \frac{\Delta}{2}\right) dz,$$

These conditions yield the system of equations

$$k = \frac{2}{3} - \frac{1}{2}(1 - \Delta/2)\Delta, \quad k = (1 - \Delta/2)\Delta,$$

leading to $k = \frac{4}{9}$ and $\Delta = \frac{2}{3}$.

Indifference conditions for the case of $\alpha = 2$ are

$$0 = k - 1 + \frac{1}{4}\int_0^1 (z+1)^2 dz + \frac{1}{4}\int_0^{1-\Delta^2} \left((1-z)^{1/2} - \Delta\right)^4 dz$$
$$- \frac{1}{2}\int_0^{1-\Delta^2} (z+1)\left((1-z)^{1/2} - \Delta\right)^2 dz,$$

$$0 = k - 1 + \int_0^1 z dz + \int_{1-(1-\Delta)^2}^{1} \left(1 - \left((1-z)^{1/2} + \Delta\right)^2\right) dz,$$

which can be expressed as

$$0 = k - \frac{5}{12} + \int_0^{1-\Delta^2} \left[\frac{1}{4}\left((1-z)^{1/2} - \Delta\right)^4 - \frac{1}{2}(z+1)\left((1-z)^{1/2} - \Delta\right)^2\right] dz, \quad (41)$$

$$0 = k - \frac{1}{2} + (1-\Delta)^2 - \int_{1-(1-\Delta)^2}^{1} \left((1-z)^{1/2} + \Delta\right)^2 dz. \quad (42)$$

To solve the integral in (41) we do a change of variable $t = (1-z)^{1/2} - \Delta$ (implying that $dz = -2(t+\Delta)dt$), which leads to

$$0 = k - \frac{5}{12} + \int_0^{1-\Delta} \left[\frac{1}{2}t^4(t+\Delta) - (2 - (t+\Delta)^2)t^2(t+\Delta)\right] dt$$
$$= k - \frac{5}{12} + \int_0^{1-\Delta} \left[\frac{3t^5}{2} + \frac{7\Delta t^4}{2} + 3\Delta^2 t^3 - 2t^3 + \Delta^3 t^2 - 2\Delta t^2\right] dt.$$

Similarly, using a change of variable $t = (1-z)^{1/2} + \Delta$, we can express (42) as

$$0 = k - \frac{1}{2} + (1-\Delta)^2 - 2\int_{\Delta}^{1} t^2(t-\Delta)dt.$$

Solving the integrals in the above expressions and rearranging, we obtain the following system of equations:

$$k = \frac{2}{3} - \frac{8}{15}\Delta + \frac{2}{3}\Delta^3 - \frac{5}{12}\Delta^4 + \frac{1}{30}\Delta^6, \quad k = \frac{4}{3}\Delta - \Delta^2 + \frac{1}{6}\Delta^4.$$

The solution for $\alpha = 2$ involves a slightly higher cutoff than under $\alpha = 1$, given by $k = 0.447 > 0.444 \approx 4/9$ (the relative difference in cutoffs is $\Delta = 0.552$).

Finally, consider any continuous distribution F over types with full support in $[0,1]$ that places a mass of $1/2 - \varepsilon$ in a small neighborhood of $w = 0$ and a similar mass in a neighborhood of $w = 1$, for arbitrarily small $\varepsilon > 0$. Since ΔU is Lipschitz continuous in all its arguments the solution to the system of indifference conditions under F is going to be in a small neighborhood of the solution under two-types. Accordingly, the solutions under this continuous F are going to be different across the two noise distributions. □

References

Abadi, Joseph and Markus Brunnermeier, "Blockchain economics," 2018. Working Paper.

Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan, "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks," *Econometrica*, 2007, *75* (3), 711–756.

Bassetto, Marco and Christopher Phelan, "Tax Riots," *Review of Economic Studies*, 2008, *75* (3), 649–669.

Bond, Philip and Kathleen Hagerty, "Preventing Crime Waves," *American Economic Journal: Microeconomics*, 2010, *2* (3), 138–159.

Carlsson, Hans and Eric van Damme, "Global Games and Equilibrium Selection," *Econometrica*, September 1993, *61* (5), 989–1018.

Corsetti, Giancarlo, Amil Dasgupta, Stephen Morris, and Hyun Song Shin, "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders," *The Review of Economic Studies*, 2004, *71* (1), 87–113.

Dai, Liang and Ming Yang, "Organizations and Coordination in a Diverse Population," 2018. Working Paper.

Diamond, Douglas W and Philip H Dybvig, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 1983, *91* (3), 401–419.

Drozd, Lukasz A. and Ricardo Serrano-Padial, "Financial Contracting with Enforcement Externalities," *Journal of Economic Theory*, 2018, *178*, 153–189.

Dybvig, Philip H and Chester S Spatt, "Adoption Externalities as Public Goods," *Journal of Public Economics*, 1983, *20* (2), 231–247.

Edmond, Chris, "Information Manipulation, Coordination, and Regime Change," *Review of Economic Studies*, 2013, p. rdt020.

Farrell, Joseph and Garth Saloner, "Standardization, Compatibility, and Innovation," *The RAND Journal of Economics*, 1985, pp. 70–83.

Frankel, David M., Stephen Morris, and Ady Pauzner, "Equilibrium Selection in Global Games with Strategic Complementarities," *Journal of Economic Theory*, 2003, *108* (1), 1–44.

Goldstein, Itay and Ady Pauzner, "Demand–deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, 2005, *60* (3), 1293–1327.

Golub, Benjamin and Stephen Morris, "Expectations, Networks, and Conventions," 2017. Working Paper.

Harsanyi, John C and Reinhard Selten, "A general theory of equilibrium selection in games," *MIT Press Books*, 1988, *1*.

Iijima, Ryota, "Iterated generalized half-dominance and global game selection," *Journal of Economic Theory*, 2015, *159*, 120–136.

Katz, Michael L and Carl Shapiro, "Network Externalities, Competition, and Compatibility," *American Economic Review*, 1985, pp. 424–440.

Leister, C. Matthew, Yves Zenou, and Junjie Zhou, "Coordination on Networks," 2018. Working Paper.

Milgrom, Paul and John Roberts, "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities," *Econometrica*, 1990, *58* (6), 1255–1277.

Morris, Stephen and Hyun Song Shin, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 1998, *88* (3), pp. 587–597.

_ **and** _ , "Global Games: Theory and Applications," in "Proceedings of the Eighth World Congress of the Econometric Society," Cambridge University Press, 2003.

_ **, Rafael Rob, and Hyun Song Shin**, "p-Dominance and Belief Potential," *Econometrica*, 1995, *63* (1), 145–157.

Obstfeld, Maurice, "Rational and Self-fulfilling Balance-of-Payments Crises," *American Economic Review*, 1986, *76* (1), 72–81.

_ , "Models of Currency Crises with Self-fulfilling Features," *European Economic Review*, 1996, *40* (3), 1037–1047.

Peski, Marcin, "Generalized risk-dominance and asymmetric dynamics," *Journal of Economic Theory*, 2010, *145* (1), 216–248.

Postlewaite, Andrew and Xavier Vives, "Bank Runs as an Equilibrium Phenomenon," *Journal of Political Economy*, 1987, pp. 485–491.

Rochet, Jean-Charles and Xavier Vives, "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?," *Journal of the European Economic Association*, 2004, *2* (6), 1116–1147.

Sakovics, Jozsef and Jakub Steiner, "Who Matters in Coordination Problems?," *American Economic Review*, 2012, *102* (7), 3439–3461.

Vives, Xavier, "Nash Equilibrium with Strategic Complementarities," *Journal of Mathematical Economics*, 1990, *19* (3), 305–321.

Zandt, Timothy Van and Xavier Vives, "Monotone Equilibria in Bayesian Games of Strategic Complementarities," *Journal of Economic Theory*, 2007, *134* (1), 339–360.