no trade theorems

[ABSTRACT
No trade theorems represent a class of results showing that, under certain conditions, trade in asset markets between rational agents cannot be explained on the basis of differences in information alone. They pose a challenge to provide a theoretical justification of the high trade volumes observed in financial markets. This article overviews existing no trade theorems and discusses alternative approaches to modelling information-based trade.
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The very high levels of daily trading activity observed in many financial markets are often attributed to speculation: agents hold different views about how much assets are worth – for instance, they may have different expectations about future prices – and these differences should lead them to trade. Rational agents typically hold distinct opinions if they privately observe different information. Thus, as this reasoning goes, the arrival of asymmetric information should induce agents to trade. No trade theorems challenge this premise by showing that, if the initial asset allocation is commonly known to be efficient, then any proposed trade after the arrival of new information cannot lead to a Pareto improvement over the initial allocation as long as traders interpret information in a similar fashion. As a consequence, agents do not have an incentive to trade after receiving the new information.

The logic behind these results goes as follows. Consider the market of an asset in which it is common knowledge, before the arrival of new information, that the initial allocation is efficient. This implies that the asset is allocated to the agents that value it the most. Otherwise, there would be a mutually beneficial trade in which some agents holding units of the asset sell them to agents with higher valuations, contradicting the fact that the initial allocation is efficient. With the arrival of new information, some agents may value the asset more than those holding it if the former receive a more positive signal about the asset value than the latter *and* if agents only consider their own signal when updating their beliefs about the asset. Thus, without considering any additional information, they could get to a better allocation by trading. However, the fact that an agent is willing to acquire units of the asset at a given price conveys some information about the signal she received. Since traders interpret signals in a similar fashion, this *additional* information should be taken into account by the potential sellers, who then revise upwards their beliefs about the asset and become unwilling to trade at the proposed price, even though they would have agreed to trade had they only considered their own signals.

No trade theorems have proven to be a major hurdle not only in the modelling of information-based trade, but also in analysing other aspects of trade under incomplete information, such as information aggregation or information acquisition in markets. In particular, they highlight a well-known paradox associated with the efficient markets hypothesis (Grossman and Stiglitz, 1980): if market prices reflect all the relevant information possessed by agents about asset values, traders sharing common prior beliefs cannot take advantage of their private information and thus have no incentive to acquire it in the first place.

Theoretical results

There are three basic elements in no trade theorems: efficiency of the initial allocation, common knowledge of the institutional and informational environment, and some degree of agreement in they way new information should be interpreted. Efficiency of the initial allocation implies that, before the arrival of information, there is no alternative allocation that Pareto dominates it. Common knowledge requires that agents have correct beliefs about the beliefs of others and about their equilibrium behavior and all this is commonly known by all agents. Finally, traders need to exhibit some similarities in the way they interpret new information. The strongest assumption implies agents having common priors about the distribution of asset values and private information. In this context, rational agents cannot 'agree to disagree' (Aumann, 1976; Geanakoplos, 1994), i.e. they cannot hold different beliefs after observing the same information. Weaker notions include noisy versions of concordant beliefs (priors about the value of the asset may differ but traders share the same beliefs about the distribution of information conditional on asset values) and consistent beliefs (receiving the same information leads to the same posterior beliefs).

The gist of no trade theorems is to show that efficiency of the allocation before the arrival of new information leads to efficiency of the same allocation after agents receive new information. Accordingly, the stronger the notion of efficiency, the stronger the compatibility requirements on agents' beliefs (see Holmstrom and Myerson (1983) for a classification of efficiency notions). It turns out that the relevant type of efficiency depends on the notion of market equilibrium, which establishes which types of trade are considered feasible. Most no trade theorems focus on three different equilibrium notions: common knowledge trade; incentive compatible trade; and rational expectations equilibria. Common knowledge trade refers to the case in which agents do not behave strategically, trades are public, and markets are complete (i.e. there exist a complete set of Arrow–Debreu securities), so that trades contingent on the true state of the world are possible. In this context, it will be common knowledge for rational traders that when a public trade is carried out it is because it is feasible and mutually beneficial. If agents behave strategically feasible trades are those that induce agents to truthfully reveal their private information (incentive compatible trade). Finally, rational expectations equilibrium refers to the case in which agents are non-strategic and trades are contingent on prices, which potentially reflect agents' private information. Morris (1994) identifies the relevant types of efficiency of the initial allocation that lead to no trade for these alternative equilibrium concepts and provides the corresponding restrictions on traders' beliefs.

Early no trade theorems (Rubinstein, 1975; Kreps, 1977; Tirole, 1982; Sebenius and Geanakoplos, 1983) focus on the common prior assumption to show that if agents are risk averse no mutually beneficial trade exists after the arrival of information. Milgrom and Stokey (1982) show that concordant beliefs are sufficient to rule out common knowledge trade. Generalising this result, Dow *et al.* (1990) show that, if the initial allocation is *ex ante* efficient with respect to prior beliefs, common knowledge trade is ruled out regardless of the nexus between prior and posterior beliefs, as long as agents are rational. On the other hand, Morris (1994) shows that consistent beliefs are associated with no trade when agents are strategic (incentive compatible trade). There are also no trade theorems dealing with the case where all information is public (Hakansson *et al.*, 1982).

One may be tempted to regard no trade theorems as theoretical artifacts without much empirical content. After all, agents face uncertainty in most asset markets about the number of participants, their beliefs and the sources of information they may have access to, rendering the common knowledge assumption too restrictive. However, as we explore ways to break no trade results it becomes apparent that speculative trade does not trivially follow from weakening the conditions underlying no trade theorems.

Alternative sources of information-based trade

There are different routes taken by the literature to elicit trade in models of asset markets under asymmetric information. The most frequent approaches either weaken the common knowledge assumption or exogenously introduce 'liquidity' in the market, i.e. make the initial allocation inefficient due to demand shocks. Other approaches allow agents to 'agree to disagree' by introducing bounded rationality. Finally, some models introduce uncertainty in the market. Here is a brief overview:

- Lack of common knowledge: There are different ways in which the common knowledge assumption can be relaxed. For instance, traders may not have common knowledge about potential disagreements over the asset value that asymmetric information creates, or they may not have common knowledge about the rationality of other traders. One way of relaxing it is to require that traders have common beliefs rather than common knowledge (see Monderer and Samet, 1989) for a definition of common beliefs), which implies that agents are not completely certain about other agents' beliefs or their rationality. In the context of common knowledge trade, Neeman (1996) shows that common belief of potential disagreements leads to trade only if rationality is not common knowledge. One interpretation of this result is that speculative trade can occur if agents exhibit some overconfidence: even if all traders are rational, some believe it is possible that other traders may not be so, or that other traders may (erroneously) think so. Another approach to relax common knowledge is to let agents interpret information in a dissimilar fashion or to introduce doubts about how to interpret information, whether public or private. Differences in interpretation of public information among bayesian agents may arise with common priors if agents additionally receive private information (Andreoni and Mylovanov, 2010) or when they hold different priors and are uncertain about how to interpret some signals (Acemoglu *et al.*, 2009).

- Demand shocks/noise traders: Many theoretical models aimed at studying information aggregation, insider trading and other interesting phenomena in financial markets get around no trade by introducing exogenous sources of liquidity, that is, positive demand/supply for the asset at any given price. This is done by either having aggregate demand shocks (Hellwig, 1980; Diamond and Verrecchia, 1981; Kyle, 1985, 1989) or by introducing agents with immediate (exogenous) liquidity needs willing to sell/buy at current prices (Glosten and Milgrom, 1985; Easley and O'Hara, 1992).

- Bounded rationality: There are many ways in which bounded rationality and psychological biases in the way agents update beliefs can elicit trade. Geanakoplos

(1989) characterizes the conditions on the information structures associated to bounded rationality under which speculative trade is possible, which basically require agents' information structures not being represented by a partition of the space of possible states of the world (see Rubinstein and Wolinsky (1990) for an example of speculative trade when information structures are non-partitional).

- Uncertainty: A potential way to elicit trade is to introduce (Knightian) uncertainty by letting agents hold multiple priors rather than a single one. In this context, Kajii and Ui (2009) show that for certain classes of preferences under uncertainty the updating rule mapping priors to posteriors is the key determinant of the existence of speculative trade. A no trade theorem still applies if the set of posterior beliefs is the collection of all conditional probability distributions of the priors (*full bayesian updating*). If, on the other hand, the set of posteriors is the collection of all conditional distributions that maximize the likelihood of the observed private information (*maximum likelihood updating*) speculative trade can happen. Dow *et al.* (1990) provide an early example of trade in the presence of uncertainty when the arrival of information completely resolves all of the initial uncertainty.

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See also **asymmetric information; common knowledge.**

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